1988

Three Essays In Mechanism Design

Dominique Michel Demougin

NOTICE

The quality of this microform is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Previously copyrighted materials (journal articles, published tests, etc.) are not filmed.

Reproduction in full or in part of this microform is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30.

AVIS

La qualité de cette microforme dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

Les documents qui font déjà l'objet d'un droit d'auteur (articles de revue, tests publiés, etc.) ne sont pas microfilmés.

La reproduction, même partielle, de cette microforme est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30.

THREE ESSAYS IN MECHANISM DESIGN

by

Dominique M Demougin

Department of Economics

Submitted in partial fulfilment
of the requirements for the degree of
Doctor of Philosophy

Faculty of Graduate Studies
The University of Western Ontario
London, Ontario
July 1988

ISBN 0-315-43305-1

ABSTRACT

Microeconomic literature has shown an increasing interest in problems with asymmetric information. A general consensus from this section of the literature is that the introduction of an asymmetry in information affects the distribution of wealth between the different agents as well as the overall efficiency of the system. It is also generally thought that a mechanism designer can do better than to average over the information set or, alternatively, that communication has a positive value. The present research shows that these conclusions are very sensitive to changes in the assumptions of the models. The first essay analyzes a principal-agent model with moral hazard and adverse selection. It is shown that for a large class of environments communication has no value and that the principal cannot do better than to average over the different types of agents. In the second essay a principal-agent model with adverse selection is considered. Furthermore, it is assumed that both parties can observe ex-post a random variable correlated to the type of the agent. It is shown that in this case the principal can support the first best solution and that in expected value the distribution of wealth is not affected by the asymmetry of information. In the last essay a heterogeneous oligopoly model with imperfect monitoring is examined. The producers are assumed to only know their own production and the prices are taken to depend stochastically on output. Again, it is shown that despite this asymmetry in information the oligopolists can sustain the cooperative quantities for a large class of environments.

ACKNOWLEDGEMENTS

The valuable assistance of Ignatius Horstmann, Preston McAfee and Philip Reny throughout this research endeavor is greatfully recognized. Comments and discussions by Arthur Fishman and Randall McArthur of some sections of these essays contributed to their final form. The Social Sciences and Humanities Resesarch Council of Canada is greatfully acknowledged for financial support.

Finally, I wish to thank my wife for her moral support without which this work would not have been possible.

TABLE OF CONTENTS

INTRODUCTION

Microeconomic analysis is largely concerned with the behavioral study of individual decision makers. One of the basic difficulties faced by decision makers, independent of the economic system in which they exist, is the decentralization of information. This fundamental problem had long been ignored. Some of the most basic results in microeconomics, such as the two theorems of welfare economics, have been derived under the assumption of a symmetric information structure.

Over the last two decades economists have started to thoroughly investigate the consequences of asymmetric information structures. When a decision maker does not observe some of the relevant data, known by other agents in the economy, he will attempt to collect some of this information. This leads to a problem of strategic behavior. An agent who holds a relevant piece of information will attempt to influence the decision process with it to his own advantage. Since the decision maker is also aware of the situation, he too will behave strategically.

A method which has been widely used, is to assume that the decision maker designs a mechanism or a contract which will maximize his objective function subject to certain informational and feasibility constraints. Two examples are the theory of agency (see the review article by MacDonald [20]) and the theory of auctions (see the review article by McAfee and McMillan [23]).

A general consensus from this section of the literature is that

the introduction of an asymmetry in information affects the distribution of wealth between the different agents as well as the overall efficiency of the system. A well known example of this result can be found in the theory of auctions. When the value of an object being auctioned is the private information of the buyers, it can be shown under fairly general conditions that the optimal auction is not efficient and that the auctioneer almost never extracts the entire rent (see McAfee and McMillan [23]). A similar result holds in the theory of agency (see Laffont and Tirole [18], Maskin and Riley [21] and Baron and Myerson [4]). It is also generally thought that a mechanism designer can do better than to average over the information set or, alternatively, that communication between the different agents has a positive value.

The purpose of the present research is to show that these conclusions are very sensitive to changes in the assumptions of the models. In the first chapter we consider a principal-agent model with moral hazard and adverse selection. We show that for a large class of environments communication has no value and that the principal cannot do better than to average over the different types of agents. In the second chapter we consider another principal-agent model - this time only with adverse selection. We also assume that both parties can observe a signal correlated to the type of the agent. We show that for a number of models the principal will be able to support the first best solution and that in expected value the distribution of wealth will not be affected by the asymmetry of information. In the last

chapter, we examine a heterogeneous oligopoly model with imperfect monitoring. We assume that the different producers see only their own production and that prices are stochastic. Again, we show that despite this asymmetry in information the oligopolists can sustain the cooperative solution for a large class of environments.

A RENEGOTIATION PROOF MECHANISM FOR A PRINCIPAL-AGENT MODEL WITH MORAL HAZARD AND ADVERSE SELECTION

1.1 INTRODUCTION

Over the last decade microeconomic literature has shown an increasing interest in principal-agent relationships. These are relationships in which a firm or a regulatory agency called the principal is a Stackelberg leader in a two person game. That is, the principal is assumed to be able to move first and to precommit himself to a decision rule usually referred to as a mechanism. The agent is normally either a worker or else a firm who deals with the principal and whose utility depends, at least in part, on the mechanism. The objective of the principal is to choose a mechanism which will maximize his objective function subject to certain informational and feasibility constraints.

A principal-agent relationship can be influenced by different problems: moral hazard, adverse selection and risk-sharing. Moral hazard arises when the action taken by the agent is not observable by the principal, and when the principal and the agent value the action differently. Adverse selection arises when the type of the agent is private information, i.e. known only to himself. Finally risk-sharing problems occur when either the principal or the agent is not risk-neutral, in which case it is to the mutual advantage of both parties to share in the burden of risk.

In this chapter we consider a principal-agent model with adverse selection and moral hazard. A number of recent papers have examined a similar environment, for example Christensen [5], Laffont and Tirole [19], McAfee and McMillan [24], Melumad and Reichelstein [28], Riordan and Sappington [34] and Sappington [35]. A general conclusion of these papers is that the principal should offer the agent a menu of contracts. The agent chooses the contract which maximizes her expected utility given her type. Laffont and Tirole [19] and McAfee and McMillan [24] show that for a large class of environments the menu contracts can be linear in output.

The main result of the present chapter is that for many environments, it is possible to dispense of a menu of contracts because we can find an optimal mechanism which does not depend on the report made by the agent about her type. The mechanism becomes relatively simple and depends only on the outcome of the production process. For these environments, communication can be said to have no value.

The present analysis should be interesting for several different reasons. Frist, the standard solution concept of "principal-agent problems with private information" has recently been criticized by a number of authors, because it requires a strong precommitment capability on the part of the principal. Indeed, the usual approach demands that after the agent reports her type, the principal requires her to behave according to a pre-agreed rule given by the mechanism. However, at this stage, it usually would be to the advantage of both

parties to renegotiate their initial agreement The reason for this is that after communication, the knowledge of the principal is significantly altered. In other words, optimal mechanisms in environments with private information are usually not renegotiation-proof (see Hart and Moore [14])

For the environments considered in this chapter, the problem can be avoided, because the principal can use a mechanism without reporting, in which case his knowledge is never altered, and renegotiation never becomes an issue.

Second, there are a number of economic problems which seem to exhibit adverse selection and moral hazard but where we do not observe communication. A prime example is the tax system of an economy. From the earlier literature, we would expect the finance departement to offer a menu of taxes, which reflects the ability and willingness to work of the different agents in the economy. This conclusion clearly contradicts what we observe in reality. The results of this chapter provide an explanation for this apparent anomaly.

The same results can be used to address a further criticism made about the principal-agent literature. In a recent article Arrow [3] noted that the mechanisms prescribed by the theory are far more complex than those actually observed. In contrast, the optimal mechanism which we derive is simple; there is no communication and the optimal sharing rule depends only on output. The papers by Holmström and Milgrom [16] and McAfee and McMillan [24] provide some alternative explanations.

Though the result is only derived for an environment with moral hazard and adverse selection, we can argue that it also extends to environments in which it is costly to control the agent's action. (In fact we would always expect it to be costly for the principal to control the agent's action, first, because it is time consuming to observe the agent's action, and second, because it requires the installment of a surveillance mechanism.) A principal might then prefer to forfeit the potentially observable but costly information and use the best contract available when the agent's action is unobservable. In ignoring the cost of building and maintaining a control mechanism, the principal-agent literature may well have biased its results toward excessively complex mechanisms.

In an article developed simultaneously, Melamud and Reichelstein [28][1)] derived similar results to those found in this chapter. Both works come to the same conclusion for the case in which output and effort can only take a finite number of states. For the more general environment in which output is continuously distributed, the results differ in that alternative restrictions on the distribution of output are examined.

The remainder of the chapter is organized as follows. In the next section, we present the model. In section 1.3, we look at a simple example and derive the main result for the case of a discrete distribution. In section 1.4, we consider the case of continuous distributions. Finally in section 1.5, we present some examples. Section 1.6 provides concluding remarks.

1.2 THE MODEL

We examine a principal-agent relationship. We suppose that the principal and the agent are risk-neutral. The agent can be of any type; type is denoted by the variable z and is assumed to belong to the interval $Z = [z_0, z_1]$. The principal perceives the agent's type as being drawn from a distribution $G(z)$ with density $g(z)$.

The principal owns a production technology that requires as its input an effort level taken by the agent. We assume that the effort α belongs to the subset $\Delta \subset R_+$. We denote with $C(\alpha, z)$ the monetary equivalent of the cost of effort to the agent of type z. Thus, a difference in type reflects an inherent difference in either the productivity of the agent or in her willingness to work. We assume that a higher type has a strictly lower cost and marginal cost of effort: $C_z(\alpha, z) < 0$ and $C_{\alpha z}(\alpha, z) < 0$. The second requirement is familiar to private information problems and is usually refered to as single crossing property (see Maskin and Riley [21]).

The functional form $C(\alpha, z)$ is common knowledge. The type, the effort level undertaken and the incurred cost of this effort are assumed to be the private information of the agent. The effort level produced by the agent and an unobservable random state of nature $\theta \in [\underline{\theta}, \bar{\theta}]$ determine a monetary outcome $x(\theta, \alpha)$ which is observed by both parties. We denote the p.d.f. of θ by $f(\theta)$. The output $x(\theta, \alpha)$ is taken to be observable by both parties. The problem of the principal is to determine how the payoff should be divided between the agent and himself. The principal determines the division in order to maximize

his expected profit.

We assume that:

(A1.1) $x(.,.)$ *is continuously differentiable and strictly increasing in its arguments.*

(A1.2) $f(\bar{\theta}) > 0$.

(A1.1) and (A1.2) are the central assumptions of the chapter. The essential requirements in (A1.1) are that output is increasing in effort and that there exists a best state of nature. (A1.2) guarantees that for every effort level the output $x(\bar{\theta},\alpha)$, which is attained in the best state of nature, has a positive density.

Using the argument of Holmström [15] and Mirrlees [29], we can eliminate the random variable θ and view x as a random outcome whose distribution function depends on the effort level produced by the agent. We denote the distribution of x by $F(x,\alpha)$, with density $f(x,\alpha)$[2]. The assumptions (A1.1) and (A1.2) imply that the support of the variable x is included in the closed bounded interval $[0,x_1(\alpha)]$, where $x_1(\alpha) = x(\bar{\theta},\alpha)$.

We could have left the dependence of the variable x on the type of the agent implicit throughout. We chose to introduce the random state of nature explicitly to give a justification to the assumption that the support of output is strictly increasing with effort[3]. (A1.2) then guarantees that x has a strictly positive density at the upper bound of its support.

We require an additional regularity condition on the conditional distribution of x:

(A1.3) *The function* $f(.,.)$ *is differentiable for all* $x \in [0, x_1(\alpha)]$ *and for all* $\alpha \in \Delta$.

The standard approach to the above problem is to search for an optimal mechanism $M = (s, \alpha)$, which is made up of a sharing rule s and an effort rule α; see for example the paper by Christensen [5]. The mechanism works as follows: the principal requires the agent to make a report r about her type. Given this report, the principal suggests to the agent that she undertake the effort $\alpha(r)$. Finally, the report together with the outcome of the variable x, determines the payment to the agent $s(x,r)$ as well as the profit of the principal $x - s(x,r)$.

The set of potential mechanisms is restricted by two constraints. First, an individual rationality constraint which states that the agent must be made at least as well off as his next best alternative which is taken to be zero. Second, an incentive compatibility constraint which requires that it always be in the interest of the agent to honestly report her type and also to supply the effort suggested by the principal. From the generalized version of the Revelation Principle, we know that this second requirement is without loss of generality (for reference, see Myerson [30]). The mechanism $M^* = (s^*, \alpha^*)$ is optimal if it maximizes the following control problem:

$$\max_{s,\alpha} \int_{z_0}^{z_1} \int_0^{x_1(\alpha(z))} \{x - s(x,z)\} f(x,\alpha(z)) \, dx \, dG(z)$$

(I)

$$(1.1) \quad \forall z \in Z, \quad \int_0^{x_1(\alpha(z))} s(x,z) f(x,\alpha(z)) \, dx - C(\alpha(z),z) \geq 0$$

$$(1.2) \quad \forall z \in Z, \quad (\alpha(z),z) = \underset{\alpha,r}{\operatorname{Argmax}} \int_0^{x_1(\alpha)} s(x,r) \cdot f(x,\alpha) \, dx - C(\alpha,z).$$

Alternatively, the mechanism may be interpreted in the following way. The principal offers to the agent a menu of sharing rules $s^*(x,z)$. The agent chooses the sharing rule which is best for her. Given this sharing rule the agent then produces the effort level which maximizes her own expected utility.

A recent article by McAfee and McMillan [24] used a similar model with two significant differences. First, they considered competition among the agents, whereas we assume that there is only one agent (alternatively, we could assume that there are many agents, but that the principal must deal with each of them; an example of this would be the optimal tax problem). Second, they assumed a fixed support for the distribution of x. In their paper, they showed that the optimal sharing rule remains the same even in the face of competition among the agents and also that under fairly weak restrictions the principal can find an optimal sharing rule which is linear in output, i.e. of the form:

$$s(x,z) = a(z) + b(z)x \ .$$

The present chapter addresses a different issue. As mentioned, we do not consider competition among the agents. This is significant, because in the case of competition, communication between the principal and the agents serves a dual purpose: first, it makes it possible for the principal to select an agent on the basis of her report, and second, it fixes the sharing rule. In the present case communication serves solely to select the sharing rule appropriate to the agent's type.

In this context, communication (from the agent to the principal) is said to have no value if there exists a mechanism $\hat{M} = (\hat{s},\hat{\alpha})$ which optimizes program (I) and where $\hat{s}$ is a function of x only. In such an environment, the mechanism and the incentive compatibility constraint have a slightly different interpretation. When the principal communicates to the agent the mechanism, he suggests to her that she undertakes the effort $\alpha(z)$ if her type is z. The mechanism is then incentive compatible if it is always in the agent's own interest to follow this suggestion.

Suppose that $M^* = (s^*,\alpha^*)$ is an optimal mechanism. If we consider the optimization problem of the agent, it becomes apparent that we can separate her reporting decision from her decision about an optimal effort level. Indeed:

$$\max_{\alpha,r} \int_0^{x_1(\alpha)} s^*(x,r)\ f(x,\alpha)\ dx \ - \ C(\alpha,z)$$

$$= \underset{\alpha}{\text{Max}} \; \{ \underset{r}{\text{Max}} \int_0^{x_1(\alpha)} s^*(x,r)\, f(x,\alpha)\, dx \} \; - \; C(\alpha,z) \quad .$$

Define:

$$(1.3) \qquad \tau(\alpha) = \underset{r}{\text{Max}} \int_0^{x_1(\alpha)} s^*(x,r)\, f(x,\alpha)\, dx$$

$\tau(\alpha)$ denotes the expected transfer to the agent when she produces the effort level α and, given this effort makes a report which maximizes her expected rent. Using this notation, the optimization problem of the agent with respect to effort becomes:

$$\underset{a \in \Delta}{\text{Max}} \quad \tau(a) - C(a,z)$$

It should be apparent that communication has no value to the principal if there exists a sharing rule $\hat{s}(x)$ such that:

$$\forall z \in Z \;, \quad \forall\, \alpha \in \Delta, \quad (1.4) \quad \tau(\alpha) = \int_0^{x_1(\alpha)} \hat{s}(x)\, f(x,\alpha)\, dx$$

Indeed, when (1.4) is solvable, whether the principal uses the mechanism $M^* = (s^*,\alpha^*)$ or $\hat{M} = (\hat{s},\alpha^*)$, his expected profit and the expected rent of an agent of type z are the same for every type. To prove this statment, just notice that the expected transfer of an agent of type z is the same for every possible effort level α and every type. Therefore, since M^* is incentive compatible, $\hat{M}$ must also be incentive compatible. The solvability of (1.4) clearly

depends on $r(\cdot)$, $F(\cdot,\cdot)$ and $x_1(\cdot)$. The main result of the chapter shows that there exists a set of assumptions and regularity conditions which ensure that equation (1.4) is solvable.

The standard approach, which was described earlier, has recently been criticized by a number of authors (e.g. Hart and Moore [14]) for the following reason. The solution to problem (I) yields a mechanism which is incentive compatible and simultaneously yields an effort function which is not ex-post efficient[4)] (see theorem 1.2). Since the mechanism will induce the agent to reveal her true type, after reporting both parties will have the same knowledge. Therefore, it would be advantegeous, at this point, for both parties to renegociate their initial agreement because of the inefficiency of the mechanism. The problem then is, that there is nothing endogeneous to the model which can prevent the two parties from renegociating, however, to allow renegotiation would break the initial mechanism, since the agent would anticipate it and, would, therefore, not reveal her true type.

In an environment where communication has no value, the difficulty created by potential renegotiation can be avoided because there exists a mechanism with no reporting. For this mechanism there can be no precommitment difficulty on the part of the principal because his knowledge is never altered. A mechanism of this type is said to be renegotiation proof meaning that it never is to the advantage of both parties at any time during the game to rewrite the initial agreement.

1.3 THE DISCRETE CASE

Throughout this section, we assume that output and effort can only take a finite number of values. More precisely, we denote the set of possible outputs X and the set of attainable efforts Δ and assume: $X = \{x_1, \ldots, x_n\}$, $\Delta = \{\alpha_1, \ldots, \alpha_m\}$.

Finally denote with:

$$P_{ij} = \text{prob}[X=x_i \mid \alpha=\alpha_j] \; , \; P = [P_{ij}]_{\substack{i=1,\ldots,n \\ j=1,\ldots,m}}$$

In this discrete environment, the analog of (1.4) states that communication has no value to the principal if, for the vector of expected transfer $\tau(\Delta) = [\tau(\alpha_1), \ldots, \tau(\alpha_m)]$, there exists a vector of payment $s(X) = [s(x_1), \ldots, s(x_n)]$ such that

$$(1.4') \qquad \tau(\Delta) = P \cdot s(X)$$

That is, communication has no value to the principal if the vector $\tau(\Delta)$ is in the image set $P(R^n)$. One way to guarantee that (1.4') is solvable is to require that every vector in R^m be in the image set $P(R^n)$, or, equivalently, that $P(R^n) = R^m$, this proves the following result (the same conclusion has been derived by Melumad and Reichelstein [28]):

Lemma 1.1: *In the discrete environment described above, a sufficient condition for communication to have no value is that* $P(R^n) = R^m$.

The condition of the lemma is obviously very restrictive because

we require that (1.4′) be solvable for every possible expected transfer vector. We can easily find examples where communication has no value though the condition of lemma 1.1 is not satisfied. A simple example will suffice. Assume $X = \{x_1, x_2\}$ and $\Delta = \{\alpha_1, \alpha_2, \alpha_3\}$ with

$$P^T = \begin{bmatrix} 1/2 & 1/2 \\ 1/3 & 2/3 \\ 1/3 & 2/3 \end{bmatrix}$$

Assume that for every type z, the costs of producing the effort level α_3 are larger than those of α_2. Accordingly, any optimal mechanism will require $\alpha(z) \in \{\alpha_1, \alpha_2\}$. Say $\tau(\Delta) = [\tau(\alpha_1), \tau(\alpha_2), \tau(\alpha_3)]$ is the optimal expected transfer. Then $\tilde{\tau}(\Delta) = [\tau(\alpha_1), \tau(\alpha_2), \tau(\alpha_2)]$ leads to the same effort and the same expected transfer. But obviously $\tilde{\tau}(\Delta) \in P(R^2)$. Thus, despite the fact that the condition of lemma 1.1 is not satisfied, communication has no value to the principal in this example.

1.4 THE CONTINUOUS CASE

The discrete environment which we considered in the foregoing section led in a straightforward fashion to a nice and general result. We might have expected the same for a continuous distribution; unfortunately this is not the case. As before, let Δ denote the compact set of possible effort levels and X the compact set of possible monetary outcomes. Using standard notation, $C(\Delta)$ and $C(X)$

denote respectively the sets of continuous bounded functions on Δ and on X. A mapping from the set $C(X)$ into the set $C(\Delta)$ is called an operator. Using these definitions, it can now be seen that equation (1.4) defines a particular operator. This mapping is referred to in the mathematical literature as a Fredholm operator of the first kind (in honor of the Swedish mathematician who first analyzed this particular type of equation[5]).

$$\Lambda : C(X) \rightarrow C(\Delta)$$

$$(\Lambda s)(\alpha) = \int_0^{x_1(\alpha)} s(x)\, dF(x,\alpha)$$

As in the foregoing section, communication will have no value to the principal if the operator Λ is invertible. In general, Fredholm operators of the first kind are not invertible: in fact, operators of this type belong to a class of so-called "ill-posed" problems.[6] However, because of the requirements which were imposed on the distribution of the variable x through the assumptions (A1.1)-(A1.3), Λ simplifies to an operator which can easily be transformed into a Volterra equation of the first kind[7]. Under simple regularity conditions, equations of this typs are invertible[8](see appendix 1).

Before we can derive a generalization of the foregoing result for the continuous case, we need to consider under which conditions the function $r(.)$, defined by equation (1.3), is differentiable. It was noted earlier that the reporting and the effort decision of the agent can be separated. This observation can now be applied in order

to reformulate the optimization problem of the principal using the expectation of the transfer rather than the transfer itself.

The control problem becomes:

$$\underset{\alpha(\cdot),\tau(\cdot)}{\text{Max}} \int_{z_0}^{z_1} \{\mu(\alpha(z)) - \tau(\alpha(z))\} g(z)dz$$

(II)

(1.5) $\quad \forall z \in Z, \quad \alpha(z) = \underset{\alpha\in\Delta}{\text{Argmax}}\ \tau(\alpha) - C(\alpha,z)$

(1.6) $\quad \forall z \in Z, \quad \pi(z) = \tau(\alpha(z)) - C(\alpha(z),z) \geq 0$

$$\text{where} \quad \mu(\alpha) = \int_0^{x_1(\alpha)} x\ f(x,\alpha)\ dx$$

In this notation, $\pi(z)$ denotes the rent which the agent will extract if she is of type z, and $\mu(\alpha)$ is the expected value of output if the agent undertakes the effort α. The solution of (II) will solve the original problem if the principal can find a transfer function which induces τ.

Using the same notation as McAfee and McMillan [24], we define:

(1.7) $\quad A(\alpha,z) \doteq \mu(\alpha) - C(\alpha,z) + \dfrac{1 - G(z)}{g(z)} C_z(\alpha,z)$

From McAfee and McMillan [24], we know that for mechanisms which induce the agent to reveal her true type, A represents the expected

net return to the principal.

Theorem 1.2: *Assume that A is twice differentiable and also satisfies the following requirements:*

(A1.4) $\forall\ z \in [z_0, z_1],\quad \exists\ \alpha \in \Delta$ s.t. $A_\alpha(\alpha, z) = 0$

(A1.5) $\forall\ z \in [z_0, z_1],\ \forall\ \alpha \in \Delta$

(i) $A_{\alpha\alpha}(\alpha, z) < 0$

(ii) $A_{\alpha z}(\alpha, z) > 0$

then, first, the following functions maximize problem (II):

$$(1.8) \qquad \alpha^*(z) = \operatorname*{Argmax}_{\alpha \in \Delta} A(\alpha, z)$$

$$(1.9) \qquad r^*(\alpha) = \pi(\alpha^{*-1}(\alpha)) + C(\alpha, \alpha^{*-1}(\alpha)) ,$$

and second, these functions are twice differentiable in their respective arguments.

(A1.4) is necessary to ensure that the effort function can be defined by an Euler equation. (A1.5)(i) guarantees that the effort function is differentiable. (A1.5)(ii) is familiar in private information problems, and implies that the effort function is strictly increasing in types (for a discussion of this last assumption, see the work by Maskin and Riley [21]).

Proof: The first section of the proof uses a first order approach. This section is directly taken from McAfee and McMillan [24].

Using the first envelope theorem, the maximization problem of the agent yields the following condition:

$$(1.10) \qquad \pi_z(z) = -C_z(\alpha(z),z)$$

but since at the optimum we also have

$$(1.11) \qquad r(\alpha(z)) = \pi(z) + C(\alpha(z),z)$$

we can rewrite the maximization problem of the principal as follows:

$$(\text{III}) \qquad \begin{aligned} &\max_{\alpha(\cdot)} \int_{z_0}^{z_1} (\mu(\alpha(z)) - \pi(z) - C(\alpha(z),z))g(z)dz \\ &\text{s.t.} \quad \pi_z(z) = -C_z(\alpha(z),z) \\ &\qquad\quad \pi(z) \geq 0 . \end{aligned}$$

Since by assumption $C_z < 0$, we know from the incentive compatibility constraint that $\pi(\cdot)$ is increasing with type. We can use this observation to eliminate the individual rationality constraint along the path and only require $\pi(z_0) \geq 0$. In substituting problem (III) for problem (II) we have ignored the second-order condition of the agent's maximization problem. We will return to this point at the end of the proof to show that when the assumptions of the theorem hold α^* is globally incentive compatible. As in Baron and Myerson [4], we can now use the first-order envelope condition to substitute $\pi(\cdot)$, indeed:

$$(1.12) \qquad \int_{z_0}^{z_1} \pi(z)g(z)dz = -\pi(z)(1-G(z))\Big|_{z_0}^{z_1} + \int_{z_0}^{z_1} \pi_z(z)(1-G(z))dz$$

Using this substitution and (1.7), problem (III) becomes:

$$\text{(IV)} \qquad \underset{\alpha(\cdot)}{\text{Max}} \int_{z_0}^{z_1} A(\alpha(z),z)\, g(z)\, dz \; - \; \pi(z_0)$$

$$\text{s.t.} \quad \pi(z_0) \geq 0 .$$

This proves that the optimal effort level is defined by (1.8). (A1.4) guaranties that (1.8) is always solvable. Furthermore, applying the implicit function theorem twice, (A1.5)(1) implies that α^* is twice differentiable. The rent which the agent will extract follows from (1.10) by integration:

$$\text{(1.13)} \qquad \pi^*(z) = \int_{z_0}^{z} -C_z(\alpha^*(\hat{z}),\hat{z})d\hat{z} + c$$

Since $\pi(z_0)$ enters negatively into the objective function of the principal, we know that he will optimally set $\pi^*(z_0)=0$. Therefore, the constant term in (1.13) is zero. In order to prove (1.9), we must show that $\alpha^*(\cdot)$ is invertible. We do this by showing that the effort level is strictly increasing with type. Using the implicit function theorem, we obtain:

$$\text{(1.14)} \qquad \alpha^*_z(z) = A_{\alpha z}(\alpha^*(z),z)/A_{\alpha\alpha}(\alpha^*(z),z) \; > \; 0$$

The assumptions of the theoerm imply that $\alpha^*(\cdot)$ is invertible and (1.9) follows immediately from (1.11). From (1.9) and (1.13) and the fact that α^* is twice differentiable, we also conclude that π^* is twice differentiable.

Finally, to conclude the proof, we must show that the second-order condition of the reporting problem of the agent is satisfied along the path $\alpha^*(z)$. To show this, take equation (1.9) and totally differentiate both sides twice with respect to effort. Rearranging the terms, we obtain:

$$(1.15)\quad [\pi_{zz}+C_{zz}]\left(\frac{\partial\alpha^*}{\partial z}\right)^{-2}+[\pi_z+C_z]\left(\frac{\partial^2\alpha^*}{\partial z^2}\right)^{-1}+2C_{az}\left(\frac{\partial\alpha^*}{\partial z}\right)^{-1} = r_{aa}(\alpha^*)-C_{aa}(\alpha^*,z)$$

This equation can be greatly simplified. From (1.10), we know that the second term of (1.15) is zero. Also differentiating (1.10) with respect to z yields the following equality:

$$(1.16)\quad \pi_{zz} + C_{zz} = -C_{az}\left(\frac{\partial\alpha^*}{\partial z}\right)$$

Substituting this equation into (1.15) yields:

$$(1.17)\quad r_{aa}(\alpha^*(z)) - C_{aa}(\alpha^*(z),z) = C_{az}(\alpha^*(z),z)\left(\frac{\partial\alpha^*}{\partial z}\right)^{-1} \le 0$$

The left hand side of (1.17) is just the second-order condition of the agent's problem. It is negative since by assumption $C_{az} < 0$, and from equation (1.11) $a_z^* > 0$.

This concludes the proof.

Q.E.D.

Using the solution of problem (II), the principal can derive the transfer function of the mechanism by solving equation (1.3) or alternatively, if communication has no value, by solving equation

(1.4). We conclude this section by presenting a set of conditions which guarantee that communication has no value.

Theorem 1.3: *If the assumptions (A1.1)-(A1.4) are satisfied the integral equation:*

$$(1.4) \qquad r(\alpha) = \int_0^{x_1(\alpha)} s(x)f(x,\alpha)dx$$

has at least one solution and communication has no value to the principal.

Proof: In (A1.1) we assumed that $x_1(\cdot)$ is an increasing function and so $x_1^{-1}(\cdot)$ exists. Denote $x_1(\alpha) = y$ and define:

$$k(x,y) = f(x,x_1^{-1}(y))$$
$$g(y) = r(x_1^{-1}(y))$$

Using this notation (1.4) can be rewritten:

$$(*) \qquad g(y) = \int_0^y s(x)k(x,y)dx$$

where $y \in [\underline{y},\bar{y}]$ and $\underline{y} = \min_{a \in A} x_1(a)$

$$\bar{y} = \max_{a \in A} x_1(a)$$

The integral equation (*) is a Volterra equation of the first kind with unknown s(.). Implicitly, the functions g(.) and k(x,.) have been assumed to satisfy a number of regularity requirements:

(i) (A1.1) and (A1.5) imply that g(.) is differentiable.

(ii) (A1.2) implies that $k(y,y) \neq 0$ for all $y \in [\underline{y},\bar{y}]$.

(iii) (A1.4) implies that $\partial k/\partial y\ (x,y)$ exists, is continuous and bounded for all $y \in [\underline{y},\bar{y}]$ and all $x \in [0,y]$.

The solution concept for an equation of this type goes back to the Italian mathematician Volterra [37]. In order to derive a solution using Volterra's method, (*) is required to be defined for all possible values of y over the interval $[0,\bar{y}]$. It is obviously feasible to extend the support of the functions g(.) and k(x,.) to this interval, such that:

- $g(0) = 0$; notice, when $\underline{y}=0$, $g(\underline{y})$ will be by construction 0.
- The requirements (i), (ii) and (iii) are satisfied over the entire support.

Given these extensions and conditions, the resulting integral equation has a unique solution. This proves that there exists at least one solution to (1.4). This solution is in general not unique because the extensions of $g(\cdot)$ and $k(x,\cdot)$ are arbitrary.

Q.E.D.

The critical requirements for the above theorem are that for all feasible effort levels the upper bound of the support has a strictly positive density - $f(x_1(\alpha),\alpha) > 0$ - and also that the support of x is strictly increasing in α. It is interesting to see how these

two assumptions translate for the environment which we examined in the foregoing section. In the case of a discrete distribution, the requirement that the support of output be moving with effort is equivalent to the assumption that the matrix of probabilities be triangular. The requirement that the upper bound of the support has a positive density implies, for the discrete case, that the matrix of probabilities has non-zero elements along its diagonal. (Note, that since in the continuous case the cardinality of X and A are equal, we consider the discrete analog where the dimension of X and A are the same.) Clearly, in such a case, the matrix of probabilities would be invertible and (1.4') becomes solvable for all possible $\tau(.)$'s.

The conditions of theorem 1.3 require that the density of output be discountinuous at the upper bound of its support. From an economic perspective, this requirement is not always appealing. Mathematically, it is also not necessary. The result of theorem 1.3 can be extended as follows:

<u>Theorem 1.4</u>: *Assume that the assumptions (A1.2), (A1.4) and (A1.5) are satisfied and that*

(A1.6)(i) $\forall \alpha \in \Delta$, $f(x_1(\alpha),\alpha) = 0$ *and* $f_\alpha(x_1(\alpha),\alpha) \neq 0$,

(ii) $f(.,.)$ *and* $x_1(.)$ *are twice continuously differentiable in effort.*

Then the integral equation (1.4) has at least one solution and communication has no value[9].

Proof: As in the proof of theorem 1.3, we can transform (1.4) into the integral equation (*). Using (A1.6) and differentiating (*) with respect to y yields:

$$(*') \qquad g'(y) = \int_0^y s(x)k_y(x,y)\,dx$$

This is again a Volterra equation of the first kind. By construction, this equation satisfies all the requirement for solvability as formulated in appendix 1.

Q.E.D.

1.5 EXAMPLES

In the first part of this section we will consider two different examples. The first example is a direct application of Theorem 1.2 and 1.3, while in the second example, we will show that even when the support of x is independent of α, we might still be able to solve the problem.

Consider the problem of a landowner who wants to rent some of his fields to a farmer. In this context, $x_1(\alpha)$ will denote the maximal quantity which the farmer can expect, if he supplies the effort level α. For this framework $x_{1\alpha}(.) > 0$ is a very natural assumption. For simplicity we suppose that $x_1(\alpha) = \alpha$. Obviously, a number of non-controllable random factors such as weather conditions, soil conditions, etc. ... also affect the crop. We assume that the sum of these non-controllable random factors can be summarized by viewing the

crop production as a random variable. For simplicity, the crop output is taken to be uniformly distributed over its support. Since the upper bound of the support depends on the effort level, we have:

$$(1.16)\quad f(x,\alpha) = \begin{cases} 1/\alpha & \forall\ x \in (0,\alpha] \\ 0 & \text{otherwise} \end{cases}$$

Finally, let us assume that the farmer can be of different types, perhaps reflecting his willingness to work. The types are supposed to be uniformly distributed over the interval [0,1]. The cost of effort for a farmer of type z is assumed to be reflected by a quadratic function:

$$(1.17)\qquad C(\alpha,z) = \frac{z\alpha^2}{4}$$

This environment satisfies all the conditions of theorem 1.2. Therefore we know that communication has no value to the landowner. The problem remains to find a crop-sharing rule which will induce the farmer to supply the optimal quantity of effort. Using the notation of the foregoing section, the expected value of the crop production, as a function of the effort supplied, is:

$$(1.18)\qquad \mu(\alpha) = \frac{\alpha}{2}$$

For this particular environment, the equivalent of programming problem (IV) takes the following form:

$$\max_{\alpha} \int_0^1 \frac{\alpha}{2} - \frac{z\alpha^2}{2}\, dz$$

From the first-order condition of the problem, we obtain the optimal effort level which the farmer will should supply as a function of his type:

$$(1.19) \qquad \alpha^*(z) = \frac{1}{2z}$$

Using the same notation as in the foregoing section, we denote with $\pi(z)$ the rent of the farmer as a function of his type. From the first-order envelope theorem we have:

$$(1.20) \qquad \pi_z(z) = - \frac{\alpha^*(z)^2}{4} = - \frac{1}{16z^2}$$

Integrating with respect to z yields the rent which the farmer will extract (note that $\pi(1) = 0$):

$$(1.21) \qquad \pi(z) = \frac{1}{16z} - \frac{1}{16}$$

Using this result and the definition of the rent of the agent, as transfer minus cost of effort, we finally obtain the optimal expected transfer function:

$$(1.22) \qquad t(\alpha) = \frac{1}{4}\alpha - \frac{1}{16}$$

Finally, the landowner must find a crop-sharing rule $s(x)$ which will induce the above expected transfer rule. From Theorem 1.2, we know that there exists such a function. It must solve the following

integral equation:

$$(1.23)\qquad \frac{1}{4}\alpha - \frac{1}{16} = \int_0^{\alpha} s(x)\left(\frac{1}{\alpha}\right)dx$$

Substitution shows that the following crop sharing rule solves the landowner's problem:

$$(1.24)\qquad s(x) = \frac{x}{2} - \frac{1}{16}$$

This example yields a particularly simple sharing rule; basically the landowner and the farmer divide the output equally between themselves (plus/minus a lump sum of 1/16).

The next example violates two of the assumptions made earlier. We will assume that the support of x is unbounded and invariant to changes in the effort supplied. The main purpose of this example is to show that there are many cases in which the hypotheses of Theorem 1.2 are not satisfied, yet the main results of the analysis still remain valid. Assume that the c.d.f. of x is given by a Poisson distribution:

$$(1.25)\qquad F(x,\alpha) = 1 - \exp(-x/\alpha) \qquad \forall x \in [0,\infty]$$

where $\lambda = 1/\alpha$ is the Poisson coefficient. The expected value of output for a particular effort level is easily calculated:

$$\mu(\alpha) = \int_0^{\infty} (x/\alpha)\exp(-x/\alpha)dx$$

$$\mu(\alpha) = -x \exp(-x/\alpha) \Big|_0^{\infty} + \int_0^{\infty} \exp(-x/\alpha)dx$$

$$(1.26)\quad \mu(\alpha) = -\alpha \exp(-x/\alpha) \Big|_0^{\infty} = \alpha .$$

The second equality follows from integration by part. In this equality, the first term on the right hand side is zero. Indeed, from l'Hospital's rule, we have:

$$\lim_{x\to\infty} \frac{x}{\exp(x/\alpha)} = \lim_{x\to\infty} \frac{1}{\frac{1}{\alpha}\exp(x/\alpha)} = 0$$

Finally, let us suppose that the cost of effort can be represented by the same function as in the previous example. The optimization problem of the principal now becomes:

$$\underset{\alpha(\cdot)}{\text{Max}} \int_0^1 \left(\alpha - \frac{1}{2} z\alpha^2\right)dz$$

The optimal effort level which the principal would like to induce, follows from the first order condition:

$$(1.27)\quad \alpha^*(z) = \frac{1}{z}$$

Using $\alpha^*(\cdot)$ to derive the expected rent and transfer function, we obtain:

$$(1.28) \qquad \pi(z) = \frac{1}{4}\frac{1}{z} - \frac{1}{4}$$

$$(1.29) \qquad r(\alpha) = \frac{1}{2}\alpha - \frac{1}{4}$$

The problem is now reduced to finding a sharing rule s(x) which satisfies the integral equation:

$$(1.30) \qquad \frac{1}{2}\alpha - \frac{1}{4} = \int_0^\infty s(x)\, \frac{\exp(-x/\alpha)}{\alpha}\, dx$$

Using the calculation which we derived in (1.26), it can be seen that the following sharing rule satisfies the above equation:

$$(1.31) \qquad s(x) = \frac{x}{2} - \frac{1}{4}$$

To conclude this section, we present a geometrical example where communication has a positive value. For this purpose, assume that the distribution of output is such that for two different effort levels a_1, a_2 :

$$\forall\, x \in X, \quad f(x,a_1) = f(x,a_2)$$

This assumption clearly violates the requirements of theorem 1.2, since $x_1(.)$ cannot be monotone. More precisely, assume that the shapes of $\mu(a)$, $r(a,z_1)$ and $r(a,z_2)$ are given by Fig. 2, where:

$$r(a,z) = C(a,z) + C_z(a,z)\,\frac{G(z)}{g(z)}$$

From the diagram, we can read that the principal will want to induce $a^*(z_1) = a_1$ and $a^*(z_2) = a_2$. We can also see that $r(a_1) \neq r(a_2)$. But since for any sharing rule $s(x)$:

$$\int_X s(x)f(x,a_1)dx = \int_X s(x)f(x,a_2)dx$$

(1.4) cannot be solvable. Thus, for the presupposed environment, communication has a positive value.

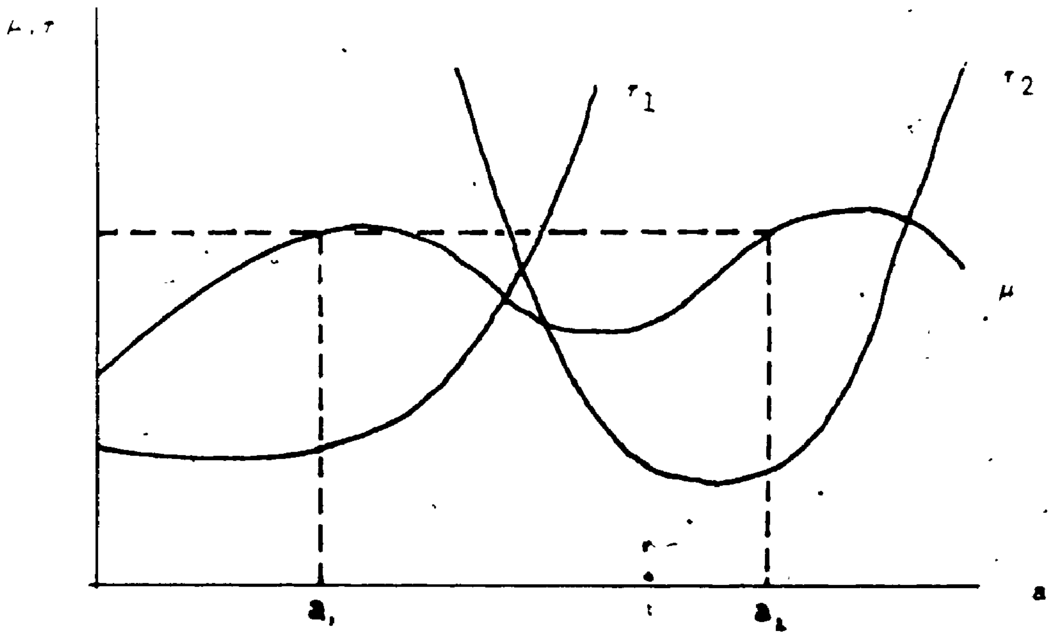

Fig.2

1.6 CONCLUSION

The main result of this chapter is that for a large class of principal-agent models with moral hazard, adverse selection and no competition for the contractors, the optimal mechanism has no need to

depend on reporting. Indeed, there exists an optimal sharing rule which depends only on the production outcome. This mechanism is obviously renegotiation proof. Also, the simplicity of the sharing rule is attractive, in particular, in the light of the recent critique by Arrow [3].

We believe that the results of this chapter have implications for many economic problems. One example which we discussed and which appears quite often in the literature is sharecropping. In this problem, it is clear that the efforts taken by a landworker are potentially observable, but that a mechanism made to observe these efforts would be costly. Given this observation and using the above result, it becomes plausible that the simple crop-sharing rules, which we observe in reality, are in fact optimal mechanisms.

More importantly, the above results have implications for the tax problem. We can view the fiscal authority as a principal, the taxpayers as agents and the search for an optimal tax structure as a principal-agent problem. This problem will obviously exhibit both moral hazard and adverse selection. Also, there are no difficulties of competition for the taxpayers, since the principal will want to tax every agent. Thus, the structure of the tax problem fits exactly the model used in this chapter. The conclusion is that the optimal tax mechanism will not require communication between the taxpayers and the fiscal authority.

FOOTNOTES

1) This research is independent from [28].

2) The model can easily be made more general. We could assume that the distribution of x is given by a function of the type F(x,h(a,z)). In this context h can be interpreted as "efficiency". Under weak regularity conditions on the cost of effort, such a model would be equivalent to the case which we analyze. For a discussion of this assumption see [24].

3) There is a misconception that a moving support will always sustain the first best solution. The first best solution can be supported in examples like Gjesdal [12] because if the agent lies there is a positive probability that he will be discovered. In which case a strategy which inflicts an infinite punishment when the agent can be proven to have lied always guarantees truth revealing. A strategy of this type does not work here even though the principal can detectsome form of deviations. The reason is that the principal cannot detect deviations when the agents undertakes less effort than he is supposed to and only deviations of this type are of interest to the agent.

4) The principal could support a mechanism which is efficient, incentive compatible and individually rational, but, in general, such a mechanism will not be optimal from his perspective. The reason is that it leaves the agent with a strictly positive rent. In which case, it becomes advantegeous for the principal to trade off some efficiency

in order to reduce the rent of the agent.

5) Fredholm, I. Swedish mathematician who worked in the early 1900. Fredholm [10] is a well known reference of his work on integral equations.

6) Let K be an operator $K: X \to Y$ and consider an equation $g = Ku$. The problem is said to be ill-posed if any of ~~the~~ conditions do not hold.

(a) $\forall g \in Y$, there exists $u \in X$ s.t. $g = Ku$

(b) The solution u is unique in X.

(c) The dependence of u upon g is continuous.

7) Volterra, V. Italian mathematician who worked at the end of the nineteen and early twenty century. Volterra and Pérès [38] include most of the work by Volterra on integral equations.

8) While we restrict the scope of the p.d.f. $f(x,\alpha)$ in order to solve for (1.4) exactly, [28] defines a solution concept based on an ε-argument. The advantage of their approach is that it works for a larger class of distributions. Its disadvantage is of course that it does not yield an exact solution. Also, the fact that Fredholm operators of the first type can be noncontinuous weakens their solution concept.

9) The case where $f(x_1(\alpha),\alpha) = 0$ for some but not all $\alpha \in A$ is more difficult and has not yet been solved.

AN EX-POST EFFICIENT MECHANISM FOR A PRINCIPAL-AGENT MODEL WITH ADVERSE SELECTION

2.1 INTRODUCTION

In this chapter, we consider another principal-agent model, but this time only with adverse selection. Again, the objective of the principal is to choose a decision rule, which will maximize his own welfare, whereby he must take into account restrictions imposed by the agent's behavior as well as by the informational structure. Principal-agent models with asymmetric information structure can also be seen as information extraction problems. The principal designs a mechanism in order to extract the private information of the agent at a minimal cost.

A number of economic problems fit the structure of a principal agent model with asymmetric information. Some examples have already been analyzed in the recent literature. Baron and Myerson [4] reexamined the problem of regulating a monopolist. The traditional solution to this problem goes back to Dupuit [9] and Hotelling [17] and presupposes that the principal has complete information about the cost of the firm. However, as Baron and Myerson point out: "it is natural to expect that a firm would have better information regarding its cost than would a principal". Another example is found in an article by Maskin and Riley [21]. This paper considers the pricing policy of a monopolist, who does not observe a relevant parameter in

the consumer's preferences.

A feature common to these papers is that the introduction of asymmetry in information affects the distribution of wealth between the principal and the agent and introduces distortions (that is, deviations from the first best solution). In the problem examined by Baron and Myerson [4], the monopolist is shown to be able to extract a positive profit. Also, the optimal production level is less than it otherwise would have been under complete information. Similarly, Maskin and Riley [21] show that a price discriminating monopolist, who faces imperfect information, will not supply the first best quantities. The monopolist also loses some profit to the buyers.

These results are based on a crucial informational assumption. It is implicitely assumed that the two parties cannot observe ex-post any variable correlated to the private information of the agent. Here are some examples which violate this assumption:

(i) The agent is a firm and his private information affects his production cost. After production both parties observe a signal correlated to the incurred cost of the firm.

(ii) The agent is a firm which produces pollution as a by-product. The private information of the firm characterizes its technology and as such affects the quantity of pollution produced for a given output level. Both parties observe a signal correlated to the pollution produced by the firm.

(iii) As in Baron and Myerson [4], the agent is a monopolist. His private information is an input price or a variable which measures his productivity. In either case, both parties are assumed to observe an index correlated to the information of the monopolist.

The main purpose of the present chapter is to show that the standard results of the principal agent literature (that is, inefficiency and a positive rent to the agent) can be completely reversed if the principal and the agent can observe ex post a variable correlated to the agent's type. This result is obvious under perfect correlation. The contribution of this chapter is to show that it generalizes even under a relatively weak correlation.

A number of papers have dealt with a similar issue; Crémer and McLean [7,8] and Maskin and Riley [22], McAfee, Mcmillan and Reny[26] examine the auctioneer's problem when the distribution of the agent's type display correlation. The general consensus is that for a large class of distribution the auctioneer will be able to implement an efficient allocation and also extract the entire surplus. These papers concentrate mainly on existence proofs. In contrast, we provide a closed form solution to the principal's problem. McAfee and Reny [27] consider a model similar to the one studied in this chapter. The two works differ in that we examine different restrictions on the conditional distribution of the agent's type.

The remainder of the chapter is organized as follows. In the next section we present the model. In section 2.3 we derive the main

results. In section 2.4 we calculate two examples. Finally section 2.5 offers some concluding remarks.

2.2 THE MODEL

We examine a principal-agent relationship. The agent is a firm which produces an output q at the cost $C(q,\theta)$. The parameter θ characterizes the technology of the firm and belongs to the interval $[\theta_0,\theta_1]$. The production creates a by-product (e.g. pollution). The quantity of the by-product depends on the amount of output produced and the technology parameter; $p = p(q,\theta)$. For simplicity, we assume that the producer does not sell his output on a market and instead that the principal compensates the firm for the cost of production with a monetary transfer T[1]. The incurred cost of production, the quantity of the by-product and the technology parameter are assumed to be unobservable by the principal. The structure of the functions $C(.,.)$ and $p(.,.)$ are assumed to be common knowledge.

The principal is a regulatory agency with a welfare function $W(q,p) - T$. Finally, we assume that there exists a random variable x, observable ex-post by both parties, which is correlated to p.

We assume that:

(A2.1) *The principal and the agent are risk-neutral. The principal maximizes expected welfare and the agent expected rent.*

(A2.2) $\forall\ \theta \in [\theta_0,\theta_1]$, $W(q,p(q,\theta)) - C(q,\theta)$ *is strictly concave and takes a maximum in output.*

(A2.3) *The technology parameter is distributed according to the density function* $g(\theta)$ *over the interval* $[\theta_0, \theta_1]$.

(A2.4) *The random variable* x *is distributed according to the conditional density function* $f(x|p)$ *over the interval* $[x_0, x_1]$.

For some sections of the chapter we will require the distribution of x to satisfy a further assumption:

(A2.5) *The conditional distribution of* x *satisfies*:

(a1) *For all possible* p, $\forall x \notin [0, x_1(p)] \quad f(x|p) = 0$.

(a2) *For all possible* $p \quad f(x_1(p)|p) > 0$.

(a3) *For all possible* $p \quad x_1'(p) > 0$.

The model characterized by the requirements (A2.1)-(A2.4) fits the structure of a number of economic problems. In particular, the three examples which were mentioned in the introduction are a special case of these assumptions:

(i) You can obtain the first example by assuming that the welfare of the principal is independent of p and by setting $p = C$. The random variable x is then a signal about the cost of the firm. In this particular environment the assumption (A2.5) seems natural. $x_1(C)$ is the largest value of x which the principal could possibly observe when the cost of the producer are C. (a3) simply requires that this value be strictly increasing in the true cost. An example for which (A2.5) is satisfied, is to assume:

$$x = z.C$$

where z is a random variable distributed over the interval [0,1] with a strictly positive density at z = 1.

(ii) The second example is the prototype of the above model, where the variable p measures the pollution level produced by the firm. One possible interpretation for the variable x is that it is a measure of the entire pollution, but that the principal cannot distinguish between different sources of pollution. This interpretation provides another environment which can justify (A2.5). Denote with the variable y the entire pollution; it must be that y is of the form $y = p + \rho$. Finally, define $x = 1/y$. In this context the main requirement of (A2.5) will be satisfied if ρ can be viewed as a random variable distributed over the positive number with a strictly positive density at $\rho = 0$.

(iii) The last example can be modelled by setting $p(q,\theta) = \theta$ for every possible production level. In this case assumption (A2.5) seems less appropriate. Indeed, it does not appear realistic to assume that the knowledge of a particular price or of the productivity of a firm should affect the ex-post support of the distribution of an index. We will come back to this difficulty in section 3.

The method used in this chapter, to derive the optimal regulation for the firm, is based on the literature about incentive

compatible mechanisms. It involves the design of a regulatory policy, which takes into account, that the principal does not know the type of the firm. It also takes into consideration that the firm generally has an incentive to misrepresent itself.

A mechanism will be a pair of functions $M = (q,T)$ from the set of possible types and outputs into the set of possible productions and transfers. The mechanism works as follows: the principal requires the agent to make a report about his technology parameter. After the reporting, the principal requires the firm to produce according to the mapping $q(.)$. After production, the principal and the agent observe the random variable x. The report r together with the variable x determine the transfer to the producer according to the mapping $T(.,.)$. The objective of the principal is to find a mechanism that maximizes his expected welfare.

In his search for a mechanism, the principal will take into account some constraints imposed by the structure of the problem. First, the principal must consider the individual rationality constraint of the agent. It states that the agent must always be made at least as well off as his next best alternative. Second, using the Revelation Principle, the principal can restrict the search to mechanisms which are incentive compatible. Such a mechanism makes it in the agent's own interest to report honestly. For a general reference on the Revelation Principle see Myerson [31]. Accordingly, we assume that the principal searches for a mechanism $M^* = (q^*,T^*)$ which solves the following control problem:

$$\underset{q,T}{\text{Max}} \int_{\theta_0}^{\theta_1} \int_{x_0}^{x_1} \{w(q(\theta),p(q(\theta),\theta)) - T(\theta,x)\} f(x|p(q(\theta),\theta)) g(\theta) dx d\theta$$

(I)

$$(2.1)\ \forall\theta\epsilon[\theta_0,\theta_1], \int_{x_0}^{x_1} \{T(\theta,x) - C(q(\theta),\theta)\} f(x|p(q(\theta),\theta)) dx \geq 0$$

$$(2.2)\ \forall\theta\epsilon[\theta_0,\theta_1],\ \theta = \underset{r}{\text{Argmax}} \int_{x_0}^{x_1} \{T(r,x) - C(q(r),\theta)\} f(x|p(q(r),\theta)) dx$$

The models analyzed by Baron and Myerson [4] and Maskin and Riley [21] can be considered a special case of the problem considered in this chapter, where the distribution of commonly observed variable and the welfare of the principal are independent of the variable p. In this particular case, if the principal were to make the transfer to the agent depend on the outcome of the random variable x, it would amount to "pure" randomization (in this context, Baron and Myerson [4] proved that pure randomization cannot increase the welfare of the principal).

2.3 THE MAIN RESULTS

In most principal agent models with asymmetric information, the principal could support a mechanism, which is efficient and incentive compatible. However, in the standard model, like Baron and Myerson [4] or Maskin and Riley [21], an efficient mechanism cannot be optimal from the principal's perspective. The reason is, that it leaves the

agent with a strictly positive rent which means that it becomes advantageous for the principal to trade off some efficiency in order to reduce the rent left to the agent. In the present model, we will see that this argument breaks down, in general, because the principal can find a mechanism which is efficient, incentive compatible, individually rational and also extracts all the expected rent of the agent.

2.3.1 The first-order approach

Before analyzing the complete principal's problem, it is useful to examine a simplified version of program (I). In this version, we substitute the second constraint with a weaker condition. Instead of requiring of the mechanism that truth telling be a global maximum of the agent's reporting problem, we only require that it satisfies the first order condition of the agent's maximization.

It is well known that this approach is, generally, not valid because the solution to the simplified version may not be globally incentive compatible. The result of this section remains of interest because we actually construct a mechanism which maximizes the simplified version of the principal's problem. Therefore, for any particular environment the global incentive compatibility can be checked.

The simplified version of the principal's problem is:

$$\underset{q,T}{\mathrm{Max}} \int_{\theta_0}^{\theta_1}\int_{x_0}^{x_1} \{W(q(\theta),p(q(\theta),\theta)) - T(x,\theta)\} f(x,p(q(\theta),\theta)) g(\theta) dx d\theta$$

(II)

$$(2.1)\quad \forall\theta\in[\theta_0,\theta_1],\quad \int_{x_0}^{x_1} \{T(x,\theta) - C(q(\theta),\theta)\} f(x,p(q(\theta),\theta)) dx \quad \geq \quad 0$$

$$(2.3)\quad \forall\theta\in[\theta_0,\theta_1]\ \frac{\partial}{\partial r}\left(\int_{x_0}^{x_1} \{T(x,r) - C(q(r),\theta)\} f(x,p(q(r),\theta)) dx \right)\Bigg|_{r=\theta} = 0$$

A mechanism, which solves this problem, can only be efficient (that is , induce the production $q^*(\theta)$), if there exists a transfer function, which allows the principal to extract the entire rent of the agent and also to satisfy the simplified incentive compatibility constraint. Mathematically, equation (2.3) is cumbersome because it involves a derivative of the transfer function. However, because we require the rent of the agent to be null for every type, it is possible to use an alternative condition which simplifies the problem. For this purpose we denote with $\pi(\theta)$ the rent of the firm of type θ.

$$\pi(\theta) = \underset{r}{\mathrm{Max}} \int_{x_0}^{x_1} \{T(x,r) - C(q^*(r),\theta)\} f(x|p(q^*(r),\theta)) dx$$

From the envelope theorem, we know that the first order condition of the agent's reporting problem can be alternatively formulated as:

$$\pi_\theta(\theta) = \int_{x_0}^{x_1} \{T(x,\theta) f_p(x|p(q^*(\theta),\theta)) p_\theta(q^*(\theta),\theta) - C_\theta(q^*(\theta),\theta) f(x|\theta)\} dx$$

Since an efficient mechanism will only be optimal if the principal extracts all the rent of the agent for every possible technology parameter, we conclude that the mechanism $M^* = (q^*, T^*)$ solves problem (II) if and only if[2]:

$\forall\ \theta \in [\theta_0, \theta_1]$:

$$(2.4)\quad \int_{x_0}^{x_1} \{T(x,\theta) - C(q^*(\theta),\theta)\} f(x|p(q^*(\theta),\theta)) dx = 0$$

$$(2.5)\quad \int_{x_0}^{x_1} T(x,\theta) f_p(x|p(q^*(\theta),\theta)) p_\theta(q^*(\theta),\theta) dx - C_\theta(q^*(\theta),\theta) = 0$$

The next result simply states that if there is enough correlation between the commonly observed variable and the technology parameter the mechanism which solves problem (II) will be efficient.

<u>**THEOREM 2.1**</u>: *Assume that the assumptions (A2.1)-(A2.4) are satisfied. A necessary and sufficient condition for an ex-post efficient mechanism to maximize (II) is that:*

$\forall \theta \in [\theta_0, \theta_1]$, there exists a set of non-zero measure $\chi(\theta) \subset [x_0, x_1]$ s.t.

$$\forall\ x \in \chi(\theta), \quad f_p(x|p(q^*(\theta),\theta)) p_\theta(q^*(\theta),\theta) \neq 0$$

PROOF: In order to prove the theorem we construct a transfer function which solves the system of equations (2.4) and (2.5). It is useful to write the transfer function as follows:

$$T^{*}(x,\theta) = t(\theta) + s(x,\theta)$$

Note that[3]:

$$\int_{x_0}^{x_1} T^{*}(x,\theta) f_p(x|p(q(\theta),\theta))dx = \int_{x_0}^{x_1} s(x,\theta) f_p(x|p(q(\theta),\theta))dx$$

Define:

$$s(x,\theta) = \begin{cases} \dfrac{C_\theta(q^{*}(\theta),\theta)}{f_p(x|p(q(\theta),\theta))p_\theta(q(\theta),\theta) \int_{\chi(\theta)} dx} & , \quad x \in \chi(\theta) \\ 0 & , \text{ otherwise} \end{cases}$$

By construction $T^{*}(.,.)$ will satisfy equation (2.5). We now define $t(.)$ such that equation (2.4) is also satisfied:

$$t(\theta) = C(q^{*}(\theta),\theta) - E[s(x,\theta)|p(q(\theta),\theta)]$$

To prove that the condition of the theorem is necessary, suppose that there exists a θ for which the above requirement is not satisfied. For this θ, equation (2.5) can never be satisfied, since the integral is zero for any bounded T.

Q.E.D.

There are at least two important observations from the above result. First, the condition of the theorem requires that the technology parameter affects the distribution function of the commonly observed variable <u>directly</u> in a non-trivial way. Second, the assumption of risk-neutrality is very critical for the theorem. This becomes particularly apparent when the correlation between x and θ becomes weak. In particular, as the correlation approaches zero the range of $s(.,.)$ goes to infinity.

2.3.2 The complete solution

In order to completely solve the principal's problem, we need to find a set of regularity conditions for the distribution function $f(.)$, the function $p(.)$ and the cost function $C(.)$, which will guarantee that honest reporting is a global maximum of the agent's reporting problem.

In the traditional principal-agent model, with only one private information and no correlated random variable, it has been fairly easy to find economically meaningful requirements, which guarantee that the first order condition of the agent's reporting problem is only satisfied at a global maximum. There are usually two assumptions which are required. First, a condition which states that the marginal costs are monotone in the agent's type. This requirement is usually referred to as the single crossing property. Second, a condition which ensures, that output is monotone in the type of the agent (this usually involves a restriction on the inverse hazard rate). For a general

reference on these kinds of conditions, see the work by Maskin and Riley [21].

In the present case, the production requirement q^* is the first best solution and, as such is defined independently of the distribution of types. Yet it remains that no single condition on the cost function will suffice to guarantee that agent's reporting problem is globally incentive compatible. The reason is simply that the agent's type enters directly into the conditional distribution of the varaible, x.

2.3.2.1 Case 1

For expository purposes, we first consider the case where the distribution of the commonly observed variable is independent of output (this corresponds to the third example mentioned in the introduction).

THEOREM 2.2: *Assume that the assumptions (A2.1)-(A2.4) are satisfied, that the function* p(.) *does not depend on output and that there exists a function* $s:[x_0,x_1] \to R$ *which solves the following Fredholm equation of the first kind:*

$$(2.6) \quad \int_{x_0}^{x_1} s(x) f_p(x|p(\theta)) p_\theta(\theta)\, dx = C_\theta(q^*(\theta),\theta)$$

Also assume that the cost function satisfies the single-crossing property. Then there exists an ex-post efficient, incentive compatible and individually rational mechanism which extracts all the expected

rent of the agent[4].

PROOF: In order to prove this theorem, we show that there exists a separable transfer function $T^*(.)$ such that the mechanism $M^* = (q^*, T^*)$ solves problem (I).

$$T^*(x,\theta) = t(\theta) + s(x)$$

Note that the left hand side of equation (2.5) becomes:

$$\int_{x_0}^{x_1} (t(\theta) + s(x))\ f_p(x|p)p_\theta dx = \int_{x_0}^{x_1} s(x)\ f_p(x|p)p_\theta dx$$

From the assumption of the theorem, we know that there exists a function s(.) which solves equation (2.5). In order to satisfy equation (2.4), we just need to define t(.) as follows:

$$(2.7) \quad t(\theta) = C(q^*(\theta),\theta) - \int_{x_0}^{x_1} s(x)\ f(x|p(\theta))dx$$

Lastly, we need to show that M^* is globally incentive compatible. Denote:

$$\pi(r,\theta) = t(r) + \int_{x_0}^{x_1} s(x)\ f(x|\theta)dx - C(q^*(r),\theta)$$

That is, $\pi(r,\theta)$ is the expected rent of the agent of type θ when he reports r. It is well known that global incentive compatibility is satisfied if:

$$\forall\ r,\theta \in [\theta_0,\theta_1], \quad \pi_{r\theta}(r,\theta) \geq 0$$

But we have:

$$(2.8) \qquad \pi_{r\theta}(r,\theta) = - C_{q\theta}(q^*(r),\theta)q_r^*(r)$$

From (A2.2) we know that q^* is differentiable, we also know:

$$q_r^*(r) = \frac{C_{q\theta}(q^*(r),r)}{w_{qq}(q^*(r)) - C_{qq}(q^*(r),r)}$$

The definition of $q^*(.)$ guarantees that the denominator of $q_r^*(r)$ is negative, while the single crossing property guarantees that the sign of $C_{q\theta}(q,\theta)$ remains the same for all (q,θ), so that indeed $\pi_{r\theta}(.) \geq 0$.

Q.E.D.

The main reason why the theorem is interesting in the present context is because it has a simple and intuitive explanation. Supposing that the principal were to initially ignore the random variable x and make the regulatory policy depend only on the agent's report. Also supposing that the principal were to implement an efficient and incentive compatible mechanism. As was explained earlier, a standard result from the existing literature on principal-agent models with asymmetric information states that the agent would be able to extract a positive rent. Denote this rent with $\pi(\theta)$ and denote the transfer function of this mechanism with $t(\theta)$.

That is:

$$(2.9) \qquad \pi(\theta) = \max_r t(r) - C(q^*(r),\theta)$$

By construction, this mechanism is efficient, since it implements the first best production. Furthermore, the requirement, that the mechanism be incentive compatible imposes a restriction on the shape of the rent function. Indeed, from the first envelope theorem we know:

$$(2.10) \qquad \pi_\theta(\theta) \quad = \quad - C_\theta(q^*(\theta),\theta)$$

Suppose now, that there exists a function s(x) such that, for all the feasible values of θ the following equation is satisfied:

$$(2.11) \qquad \pi(\theta) \quad = \quad - \int_{x_0}^{x_1} s(x)f(x|p(\theta))dx$$

Using this result, let the principal define a new transfer function as follows:

$$T^*(x,\theta) \quad = \quad t(\theta) \ + \ s(x)$$

The new mechanism (q^*,T^*) remains efficient, individually rational and incentive compatible (note, the reporting strategy of the agent remains unchanged, since $E[s(x)|p(\theta)]$ depends on the true value of θ and not on the reported value) but, by construction, it also extracts the entire rent of the firm. The contract, which we just derived, is exactly the same as that of theorem 2.2. To see that, simply notice that a function which solves (2.6) also solves (2.11) (in order to prove this statement just differentiate both sides of (2.11) with respect to θ and substitute the left hand side using

equation (2.10)).

The above theorem is also attractive because it involves the economically meaningful single crossing restriction. This restriction has an important consequence. It implies, that the optimal production is monotonic in type and as such is invertible (in fact, we proved it in theorem 2.2; see equation (2.8)). This observation leads to the following result:

<u>COROLLARY 2.3</u>: *When the assumptions of theorem 2.2 are satisfied, communication between the principal and the agent has no value.*

In order to prove this result notice that the principal can find a regulatory policy equivalent to M^* (that is a mechanism which leads to the same production and to the same transfer), but where there is no reporting. One such mechanism would be a transfer function $\hat{T}$ from the space of production levels and x's into the space of possible transfers, such that:

$$\hat{T}(q,x) = T^*(r(q),x)$$

where r(q) is the inverse function of $q^*(.)$. This conclusion is similar to the results in the foregoing chapter and Melumad and Reichelstein [28].[5)]

To conclude this exposition, we need to consider conditions which will guarantee that (2.6) is solvable. Equation (2.6) is a Fredholm equation of the first kind. As in the foregoing chapter, we could restrict the distribution of x, for example by assuming (A2.5), in order to transform equation (2.6) into a Volterra equation of the

first kind. However, as we mentioned earlier, this does not seem to be an appropriate assumption in the case where p is independent of output. An alternative would be to follow the route chosen by Melumad and Reichelstein [28]. This would necessitate restricting $f(x|p(\theta))$ and $C_\theta(q^*(\theta),\theta)$ in order to satisfy the requirements of Picard's Theorem (see Picard [32]).

A further alternative is to examine situations where (2.6) is not solvable. McAfee and Reny [27] provide a general analysis of this case. For the remainder of the section, we generalize the result of theorem 2.2. For this purpose, we introduce a new restriction on the distribution of the variable x.

(A2.6): *There exists a function* $z : [x_0, x_1] \to \mathbf{R}$ *such that* $\bar{z}(\theta)$,

$$(2.12) \qquad \bar{z}(\theta) = \int_{x_0}^{x_1} z(x) f(x|p(\theta))dx$$

is strictly increasing and concave in θ.

(A2.6) imposes a restriction on the conditional distribution of x. Unfortunately, it is difficult to know how restrictive the above requirement is. Naturally, the question arises whether some of the standard distributions satisfy **(A2.6)**. In the appendix, we show that for $x = \theta + \tau$, where τ and θ are uncorrelated the above requirement is satisfied. In the same appendix we also prove that conditional densities which are regular members of the exponential family, satisfy (A2.6). Density functions of this type include, among others, the normal, the log-normal, the Beta and the Gamma distributions. McAfee

and Rery [27] provide some further justifications for (A2.6).[6]

The next proposition will be useful for the proof of the subsequent theorem.

LEMMA 2.4: *Suppose that function $\pi(.)$ satisfies the following requirements:*

$$\forall\ r \in [\theta_0,\theta_1], \qquad r \in \underset{\theta\in[\theta_0,\theta_1]}{\mathrm{Argmax}}\ \pi(r,\theta)$$

$$\text{and} \quad \pi(r,r) = 0, \quad \textit{then}$$

$$\forall\ \theta \in [\theta_0,\theta_1], \qquad \theta \in \underset{r\in[\theta_0,\theta_1]}{\mathrm{Argmax}}\ \pi(r,\theta)$$

PROOF: By assumption:

$$\pi(r,r) \geq \pi(r,\theta)$$

$$0 \geq \pi(r,\theta)$$

$$\pi(\theta,\theta) \geq \pi(r,\theta)$$

Which proves the lemma.

Q.E.D.

This result is useful, because under the condition of the lemma we can examine the second order condition of the optimization with respect to θ instead of those with respect to reporting.

THEOREM 2.5: *Suppose that the assumptions* (A2.1)-(A2.4) *and* (A2.6) *are satisfied and that*

$$C_\theta(.)\ ,\quad C_{\theta\theta}(.) > 0$$

then there exists an ex-post efficient, incentive compatible and individually rational mechanism which extracts all the expected rent of the agent.

PROOF: In order to prove the theorem, we need to show that there exists a transfer function $T^*(x,\theta)$ such that the mechanism $M^* = (q^*,T^*)$ solves the principal's problem. We will show that the transfer function is of the form:

$$T^*(x,\theta) = t(\theta) + s(\theta)z(x)$$

Where z is the function defined in (A2.6). If $T^*(.)$ takes the form suggested, the system of equations (2.4) and (2.5) becomes:

$$(2.13)\qquad \begin{aligned} t(\theta) + s(\theta)\bar{z}(\theta) &= C(q^*(\theta),\theta) \\ s(\theta)\bar{z}_\theta(\theta) &= C_\theta(q^*(\theta),\theta) \end{aligned}$$

Since, by assumption $\bar{z}_\theta(\theta) \neq 0$, the system is solvable. This defines $T^*(.)$. We still need to show that the mechanism M^* satisfies the global maximization constraint. As before define:

$$\pi(r,\theta) = \int_{x_0}^{x_1} T(x,r)f(x|p(\theta))dx - C(q^*(r),\theta)$$

$$= t(r) + s(r)\bar{z}(\theta) - C(q^*(r),\theta)$$

Consider the maximization of $\pi(r,\theta)$ with respect to θ. The assumptior of the theorem guarantees that $\pi(r,\theta)$ is concave in the parameter θ. This together with system (2.13) ensures:

$$\forall\ r \in [\theta_0,\theta_1]$$

$$r \in \operatorname*{Argmax}_{\theta\in[\theta_0,\theta_1]} \pi(r,\theta) \quad \text{and} \quad \pi(r,r) = 0$$

Therefore, form lemma 2.4 we know:

$$\theta \in \operatorname*{Argmax}_{r\in[\theta_0,\theta_1]} \pi(r,\theta)$$

This concludes the proof of the theorem.

Q.E.D.

It is worthwhile to examine why the above result cannot hold when x and θ are independent. Obviously, when x and θ are uncorrelated, for any function z(.), we have $\bar{z}_\theta(\theta) = 0$ for all values of θ and the system (2.13) can never be satisfied.

2.3.2.2 Case 2

In this section, we go back to the initial model and assume that the distribution of the commonly observed variable depends in a non-trivial way on the output level. In order to ensure that the second order condition of the agent's reporting problem remains satisfied we constrain the distribution of x to satisfy (A2.5). This restriction

reduces the problem of the principal to the solving of a Volterra equation of the first kind.

<u>THEOREM 2.6</u>: *Assume that the assumptions (A2.1)-(A2.5) are satisfied and that for all feasible output levels* $p(q,.)$ *is monotonic in the technology parameter. Then there exists an ex-post efficient, incentive compatible and individually rational mechanism which extracts all the expected rent from the agent.*

<u>PROOF</u>: Define:

$$\pi(r,\theta) = -(r-\theta)^2$$

For every θ, this function takes a maximum of zero at $r = \theta$. For every r, define the transfer function $T^*(.,r)$ implicitly by the following integral equation:

$$(2.14)\quad \pi(r,\theta) - C(q(r),\theta) = \int_{x_0}^{x_1(p(q^*(r),\theta))} T^*(x,r)f(x|p(q^*(r),\theta))dx$$

Assumption (A2.5) guarantees that for every r equation (2.14) is solvable (for a proof of this statement, see chapter 1). By construction the mechanism (T^*,q^*) satisfies the requirements of the theorem.

Q.E.D.

The foregoing results can also be used in context of a principal who controls two or more firms which have correlated private

information. This is a natural generalization of the auction problem with correlated values analysed by Crémer and McLean [7,8], Maskin and Riley [22] and McAfee, McMillan and Reny[26]. Assume that for a given mechanism the firms behave like non-cooperative Bayes-Nash players. If the first best production of each firm depends only on her own private information, the problem of the principal becomes almost the same as the one we just examined and the foregoing results are directly applicable. The problem becomes more complex if the first best production of a firm depends on the report of some of the other producers. The first-order condition of the agent's reporting problem still works out, but the second-order condition is affected and the results of the chapter can be shown to hold only when the correlation is not too large.

2.4 EXAMPLES

This section presents two examples. In order to obtain a closed form for the mechanisms we assume simple functional forms. The first problem is an application of the theorems 2.2 and 2.5, the second an application of theorem 2.6.

Consider the following economic problem. A principal attempts to control a monopolist. The welfare function of the principal is assumed to be linear in quantity and transfer:

$$(2.15)\quad W(q,T) = 2q - T$$

Also, suppose that the cost function of the monopolist is quadratic:

$$(2.16)\quad C(q,\theta) = \theta q^2$$

In the case of complete information the solution to the principals problem would require the monopolist to produce according to the following production rule:

$$(2.17)\quad q^*(\theta) = 1/\theta$$

Finally, we assume that θ is jointly distributed with a variable x, which, ex post, is observable by all parties. Their joint p.d.f. is:

$$(2.18)\qquad f(x,\theta) = \begin{cases} x^{\theta-1} & x \in [0,1],\ \theta \in [1,e] \\ 0 & \text{otherwise} \end{cases}$$

From this equation, we can easily calculate the conditional probability of x given θ; $f(x|\theta) = \theta x^{\theta}$. Both the cost and the distribution function satisfy the restrictions of theorem 2.5. And so, there exists at least one ex-post efficient mechanism which solves the principal's problem. In order to apply theorem 2.5 we set $z(x) = x$. The system of equations described by (2.13) now becomes:

$$(2.19)\qquad \begin{cases} t(\theta) + s(\theta)\left[\dfrac{\theta}{\theta+1}\right] = \dfrac{1}{\theta} \\ s(\theta)\left[\dfrac{1}{\theta+1}\right]^2 = \left(\dfrac{1}{\theta}\right)^2 \end{cases}$$

Notice, that indeed $\bar{z}(\theta) = \theta/(\theta+1)$ is an increasing concave function. Solving the system for the functions $t(\theta)$ and $s(\theta)$ we obtain the transfer rule:

$$T^{*}(x,\theta) = -1 + [\frac{\theta + 1}{\theta}]^2 x.$$

This is a nice result, because T^* is linear in x. Since q^* is monotone in θ, there exists an equivalent mechanism without communication. In this mechanism the principal offers to the monopolist the following transfer function:

$$\hat{T}(q,x) = -1 + (1 + q)^2 x$$

It is easily verified that this transfer schedule will induce the monopolist to produce $q^*(.)$ and will extract all his profit.

Alternatively, though it is more cumbersome, we can apply theorem 2.2. Indeed, using the particular structure of the example, the equivalent of equation (2.6) becomes:

$$(2.20) \qquad \int_0^1 s(x) \frac{\partial}{\partial\theta}(\theta \cdot x^{\theta - 1})dx = 1/\theta^2$$

One solution of this equation is simply to set $s(x) = \ln(x)$. In order to see this, it is easier to integrate both sides with respect to θ and consider the equivalent integral equation:

$$(2.21) \qquad \int_0^1 s(x)\theta x^{\theta - 1}dx = -1/\theta$$

(Notice, this equation is just the analog of (2.11) for the present environment.) Substituting on the left hand side ln(x) for s(x) yields:

$$\int_0^1 \ln(x)\theta x^{\theta-1}dx = \ln(x)\, x^{\theta}\Big|_0^1 - \int_0^1 x^{\theta-1}dx = -1/\theta$$

The above equality follows from integration by part. In the first equality, the first term on the right hand side is zero. Indeed, using a simple substitution and applying l'Hospital's rule yields:

$$\lim_{x\to 0} \ln(x)\, x^{\theta} = \lim_{y\to\infty} -y/\exp(\theta y)$$

$$= \lim_{y\to\infty} \frac{-1}{\theta\exp(\theta y)} = 0.$$

To conclude the solution, we need to find the function t(.) such that for every θ the expected rent of the agent is just zero for the report $r = \theta$. That is:

$$(2.22)\quad t(\theta) = C(q^{*}(\theta),\theta) - E[\, s(x)\, |\theta]$$

It yields: $t(\theta) = 2/\theta$. Again it is easily verified, that the mechanism $M^{**} = (T^{**}(x,\theta) = 2/\theta + \ln(x),\ q^{*}(\theta) = 1/\theta)$ is indeed efficient, incentive compatible and extracts all the rent for the monopolist.

To conclude this section, we present a second example, which will be an application of theorem 2.6. Suppose that p is the amount of pollution created by the firm which the principal wants to control. The variable x is the total amount of pollution. From the perspective of the firm x is a random variable whose distribution depends on p. We assume that:

$$(2.23)\quad f(x|p) = \exp(-x + p),\quad x \in [p,\infty)$$

The amount of pollution created by the firm depends on the technology used by the firm and on the quantity of output produced. We assume the following functional form:

$$(2.24) \quad p(q,\theta) = \theta q$$

In order to solve the principal's problem, we must find the solution to the analog of equation (2.14). Denote with $v(r,\theta)$ the lefthand side of (2.14) and rewrite the integral equation:

$$(2.24) \quad v(r,\theta) = \int_{\theta q^*(r)}^{+\infty} T^*(x,r)\exp(-x + \theta q^*(r))dx$$

In order to solve this equation differentiate equation (2.24) with respect to θ and rearrange the terms. You obtain:

$$(2.25) \quad T^*(x,r) = v_\theta(r,x/q^*(r)) - v(r,x/q^*(r))q^*(r)$$

2.5 CONCLUSION

The main conclusion of this chapter is that for a large class of economic problems the presence of private information might not have as negative of an effect, as has been suggested by the existing literature on principal agent models. We showed, that in many cases the existence of a random variable, which is correlated to the agent's type and becomes observable only after the derivation of the mechanism, would wipe out the negative impact of the asymmetric information structure

The results were derived under two important restrictions; we

did not consider either moral hazard or risk-sharing problems. Moral hazard arises when the agent must take an action, which is not observable by the principal. Two examples examined by the literature on principal-agent models are Laffont and Tirole [19] and McAfee and McMillan [24]. We do not expect, that the introduction of moral hazard would greatly affect the conclusion of the chapter when effort does not enter the conditional distribution of the commonly observed variable. In particular, the result and the heuristic of theorem 2.2 would remain unchanged. The problem is more difficult when effort affects the distribution of x. In this case we would not expect the result of this chapter to carry over immediately, because the agent can trade off lying about his type with lying about the amount of effort he produces. The conclusion of this chapter would continue to hold if the principal and the agent could commonly observe more than one variable, which are all correlated to the type of the agent.

The analysis in the case of a risk averse agent should also be interesting. It introduces a new trade off: The principal will use the correlated information to extract the rent of the agent. However, in doing so the principal also introduces some randomness in the payment schedule of the agent. If the agent is risk averse, he will demand payments, which are in expected value higher in order to ensure himself against this risk. From this, we can already conclude, that it will not be optimal for the principal to induce an ex-post efficient solution; he will rather trade off some efficiency in order to reduce the expected transfer to the agent.

FOOTNOTES

1) Throughout the chapter, we assume that the firm is only compensated by the principal. Alternatively, we could assume, that the firm sells its product on a market at a price dictated by the principal and also receives a monetary transfer. This would not affect the result of the chapter.

2) For any feasible p:

$$\int_{x_0}^{x_1} f(x|p)dx = 1$$

This implies:

$$\int_{x_0}^{x_1} C(q^*(\theta),\theta)f_p(x|p)p_\theta dx = C(q^*(\theta),\theta)p_\theta \int_{x_0}^{x_1} f_p(x|\theta)dx = 0.$$

3) Use the same argument then in 2), but substitute t(.) for C(.).

4) Theorem 2.2 would still be applicable even if equation (6) is not exactly, but approximately solvable. Suppose K is an operator. We say that the equation

$$g = Ku$$

is approximately solvable if there exists a function $\bar{g}$ uniformely close to g for which the equation is solvable. Consider an example cited in Melumad and Reichelstein [28]. Assume $f(x|p(q,\theta)) =$

$(1/\theta)\exp(-x/\theta)$. If $C_\theta(q^*(.),.)$ is continuous in θ and $\theta > 0$, then equation (6) is approximately solvable.

5) In the former chapter and in Melumad and Reichelstein [28] the principal does not extract the entire profit of the agent.

6) It ~~is~~ easy to find sufficient conditions which will guarantee that (A2.6) holds. For example assume that, there exists $\hat{x} \in [x_0, x_1]$ such that $F(\hat{x}|\theta)$ is strictly increasing and concave in θ, then we would simply choose

$$z(x) = \begin{cases} 1 & x_0 \le x \le \hat{x} \\ 0 & x \gtrsim \hat{x} \end{cases}$$

(Notice, this requirement is satisfied for $F(x|\theta) = x^\theta$, $x \in [0,1]$) Alternatively, suppose that there exists a set $\chi \subset [x_0, x_1]$ of non-zero measure such that:

$$\forall\ \theta \in [\theta_0, \theta_1],\ \forall\ x \in \chi \quad \begin{cases} \text{either} & f_\theta(x|\theta) > 0 \text{ and } f_{\theta\theta}(x|\theta) < 0 \\ \text{or} & f_\theta(x|\theta) < 0 \text{ and } f_{\theta\theta}(x|\theta) > 0 \end{cases}$$

then assumption 2 can easily be seen to be satisfied, in the first case for any $z(x)$ positive and in the second case for any $z(x)$ negative.

7) A large part of this section is taken from a recent paper by Melumad and Reichelstein [28].

AN EFFICIENT MECHANISM FOR A HETEROGENEOUS OLIGOPOLY WITH IMPERFECT MONITORING

3.1 INTRODUCTION

Historically, the behavioural analysis of an oligopoly has played an important role in economic literature. The best known solution to the oligopoly problem goes back to Cournot [6] who analyzed a single period game in the absence of any binding agreement between firms. It is well known that, in the standard model, the Cournot solution yields a total output between the monopolistic and the competitive production. It is only over the last two decades that this solution concept has been criticized for yielding misleading results, which are too close to the competitive solution. A number of authors have appealed to dynamic considerations to explain why an oligopoly might, indeed, be able to behave more like a monopoly than in the Cournot solution, even though the firms cannot enforce an agreement.

One of the first rigorous formulations of the oligopoly problem, as an infinitely repeated game, is by Friedman [11]. In this paper the author shows that, if the discount rate is not too large, the single period cooperative output can be sustained in every period as a non-cooperative solution. The argument for this result is that each firm threatens to produce, in the future, the single period Cournot output if it detects cheating on the part of any other firm. This

strategy is shown to form a Nash equilibrium. When the discount rate is not too large, the "punishment" for cheating outweighs any single period gain which the firm could obtain by deviating from the cooperative quantity (Abreu [1] has a generalization of this result). This result has two weaknesses. First, it requires that cheating be detectable. This can only happen if either output is common knowledge or price is deterministic (neither assumption is very appealing). Second, as Green and Porter [13] noticed, "the deterrent mechanisms are never observed." For the purpose of this chapter it is worthwhile to note, at this point, that though the conclusions from Abreu [1] and Friedman [11] were derived using a homogeneous oligopoly, the same results would hold in the heterogeneous case.

The next logical step was, therefore, to analyze the oligopoly problem as an infinitely repeated game, but to assume that the production of a firm is its own private information and that the market price is influenced by a stochastic component. The papers by Abreu, Pearce and Stachetti [2], Green and Porter [11] and Porter [33] examine the implication of these assumptions in the case of a homogeneous oligopoly. Abreu, Pearce and Stachetti [2] shows that, under the above restrictions and some further regularity requirements, the optimal equilibrium would require the member of the cartel to always produce only one of two quantities, which corresponds to either the best or the worst symmetric sequential equilibrium. According to this equilibrium, the firms would decide which of the two quantities to produce, simply by remembering the state and the price of the

foregoing period. For either state, the equilibrium describes two complementary sets of prices, which leads to either the cooperative or the "punishment" quantities. The alternation between the two states is then shown to follow a Markov process. An interesting aspect of this equilibrium is that, though the firms will never deviate from the optimal strategy, there are periods where the cooperative quantities cannot be sustained.

The present chapter examines a heterogeneous oligopoly, but otherwise keeps essentially the same structure, as in the paper by Abreu, Pearce and Stachetti [2]. Instead of assuming that the strategy of a firm is based on "punishment" induced by production changes, we introduce the possibility of transfers. In general, transfers could either be side payments between the individual producers or "dissipative" transfers like public donations to a charity.

The main objective of this chapter will be to show that, when the discount rate is not too large, a cartel can find a set of transfer functions which:

(i) induce each member of the Cartel to produce the cooperative solution at all times,

(ii) in all periods the transfers across the firms add up to zero,

(iii) and the set of transfers can essentially be enforced as a non-cooperative equilibrium.

The remainder of the chapter is organized as follows: In the next section, we present the one period game. In 3.3 we discuss the cooperative solution. In 3.4 we prove that the cooperative solution can be induced by an <u>enforceable</u> contract, which satisfies (i) and (ii) even when output is not observable and the demands are stochastic. Finally, in the last section we prove that in the context of an infinitely repeated game, the cooperative solution is attainable even without assuming enforceable contracts.

3.2 THE MODEL

We consider an oligopoly in which N firms produce a heterogeneous product. Without further justification we suppose that these firms compete in output. The cost function of each firm is common knowledge; $c_i(q_i)$, $i = 1,\ldots N$. The vector of market prices $p = (p_1,\ldots,p_N)$ is a random vector whose distribution $f(p|q)$ depends on the entire vector of output $q = (q_1,\ldots,q_N)$. The realization of p is supposed to be common knowledge.

We assume that:

(A3.1) *Firms are risk neutral and they maximize expected profit.*

(A3.2) $\forall\ q \geq 0$, $\forall\ i \in \{1,\ldots,N\}$

Define:

$$\hat{P}_i(q) := \operatorname*{Sup}_{p_i} \{\ p_i \in R \ |\ \ p_{-i} \in R^{N-1}\ \text{ s.t. }\ f(p_i, p_{-i}|q) > 0\ \}$$

$\forall\ q \geq 0$, *the mapping* $\hat{P}(q) = (\hat{P}_1(q),\ \ldots,\hat{P}_N(q))$ *is bounded,*

bijective and $f(\hat{P}(q)|q) > 0$.

(A3.3) *The probability density function* $f(p|q)$ *and the mapping* $\hat{P}(q)$ *are* N *times differentiable in the output vector.*

(A3.2) is the central assumption of the chapter. It requires that for every feasible output level there exists a bounded square which covers the entire support of p and for which it is true that the upper right corner $\hat{P}(q)$ has a positive density. (A3.2) also demands that the mapping $\hat{P}(q)$ be invertible. There are many meaningful environments which satisfy the conditions of (A3.2).

Example, assume N=2 and

(i) $q_i = d_i(p, \epsilon_i)$ with

$$- \partial d_i/\partial p_i<0,\ \partial d_i/\partial p_j>0,\ \partial d_i/\partial \epsilon_i>0,$$

$$- (\partial d_1/\partial p_1)(\partial d_2/\partial p2_1) - (\partial d_1/\partial p_2)(\partial d_2/\partial p_1) > 0$$

(ii) $(\epsilon_1, \epsilon_2) \in [0,1]X[0,1]$ is a random vector,

(iii) (1,1) has a positive density.

In order to see that this model satisfies (A3.2) note that for any output vector the requirements in (i) imply that $\partial \hat{P}_i/\partial \epsilon_j>0$, j=1,2.

3.3 THE COOPERATIVE SOLUTION

The payoff of firm i, given (p_i, q_i), is:

$$(3.1) \quad \pi_i(p_i, q_i) = p_i q_i - c_i(q_i).$$

The cooperative solution is the vector of output q^c = $(q_1{}^c, \ldots, q_N{}^c)$ which solves the following control problem:

$$\text{(I)} \qquad \underset{q}{\text{Max}} \int_0^{\bar{P}(q)} \left(\sum_{i=1}^{N} \pi_i(p_i, q_i) \right) f(p|q)\,dp$$

Since the objective of this chapter is not to derive conditions, which would guarantee the existence of the cooperative solution, we will assume for the remainder of this chapter that the following requirement is satisfied:

(A3.4) *Problem* (I) *has at least one solution.*

From the works by Abreu [1] and Friedman [11], it is well known that under some regularity conditions, the production vector q^c can be sustained by implicit collusion as a <u>non-cooperative equilibrium</u> in the framework of a super game and when output is common knowledge.

3.4. UNOBSERVABLE OUTPUT

In this section, we consider the single period game under the additional requirements:

(A3.5) *Production is the private information of each firm.*

(A3.6) *Firms can write <u>enforceable</u> contracts.*

Assumption (A3.6) is essential for the solution of the single period game. When we consider the super game this assumption will be relaxed. The structure of the game obtained is very similar to that

of the papers by Abreu, Pearce and Stachetti [2], Green and Porter [13] and Porter [33]. There are only two significant distinctions between these papers and the present problem. First, we are examining a single period game and secondly we are considering a heterogeneous oligopoly.

Since the vector of prices depends on the state of nature and output is not observable, a firm cannot monitor the production of its competitors. A contract between the firms will be a vector of transfers $T = (T_1, \ldots, T_N)$. These transfers can only be conditioned by the variables that are commonly observed, which, in the present case, is the vector of prices. Given any contract, the problem faced by the firms is a simple Nash game. The feasibility of the contract requires that we do not introduce money from outside of the system. That is, we must require that for any price vector:

$$(3.2) \qquad \sum_{i=1}^{N} T_i(p) \leq Q$$

We will say that an agreement satisfies the production incentive criteria if: q^c is a Nash equilibrium of the production game faced by the firms. A contract will be said to be "efficient" if it satisfies the production incentive criteria and if no money leaves the system; that is if equation (3.2) is binding for every feasible state of nature.

THEOREM 1: *There exists at least one efficient contract.*

In order to prove this result, we will implicitly define a

contract by a system of (N-1) Volterra equations of the first kind with multiple integrals. We will show that the contract is efficient and that the Volterra equations satisfy sufficient requirements for solvability.

PROOF: For $i=1,\ldots,(N-1)$ we define $T_i(\cdot)$ implicitly by the following integral equation:

$$(3.3_i) \qquad \int_0^{\bar{P}(q)} T_i(p)\, f(p|q)\, dp = \int_0^{\bar{P}(q)} 1/N\Big(\sum_{\substack{j=1 \\ j\neq i}}^{N} \pi_j(p_j,q_j) - (N-1)\pi_i(p_i,q_i) \Big) f(p|q)dp$$

Finally, define the N-th transfer function:

$$(3.4) \qquad T_N(p) = - \sum_{i=1}^{N-1} T_i(p)$$

Using this definition it is easy to see $T_N(\ )$ satisfies an equation of the form (3.3_i) for $i=N$. From this last requirement, it is also obvious that the sum of transfers will always add up to zero. To prove that the contract is efficient, we only need to show that the production vector q^c is a Nash equilibrium of the production game given T. Suppose, that all the firms, except i, decide to produce the cooperative quantity. What is the best response of firm i? Firm i maximizes its expected profit with respect to its own output q_i. The profit of a firm is the sum of its payoff plus transfer. Using the

definition of the expected transfer, it is immediate that the expected profit of a firm, is a multiple of the sum of expected profits:

$$\int_0^{\bar{P}(q)} (\pi_i(p) + T_i(p)) f(p|q)dp = \int_0^{\bar{P}(q)} 1/N(\sum_{j=1}^{N} \pi_j(p)) f(p|q)dp$$

Using the usual notation, the problem of firm i, when each of its competitors produce the cooperative quantity, becomes:

$$(\mathrm{II}_i) \qquad \operatorname*{Max}_{q_i} \int_0^{\bar{P}(q^c_{-i}, q_i)} 1/N(\sum_{\substack{j=1 \\ j\neq i}}^{N} \pi_j(p_j, q^c_j) + \pi_i(p_i, q_i))\ f(p|q^c_{-i}, q_i)dp$$

By construction q_i^c solves this problem.

Finally, to conclude the proof, we must show that the integral equations (3.3_i) $i=1,\ldots,(N-1)$ are all solvable. For this purpose, let us rewrite the vector $\bar{P}(q) = y$ and denote the right-hand side of equation (3.3_i) by $v_i(q)$. Given this notation, equation (3.3_i) can be written in the standard form of a Volterra equation of the first kind with multiple integrals:

$$(3.5i) \qquad \int_0^{y} T_i(p)\ g(p|y)\ dp = t_i(y)$$

where $g(p|y) := f(p|\bar{P}^{-1}(y))$

$t_i(y) := v_i(\bar{P}^{-1}(y))$.

We need to show that each of the above equations satisfy the necessary requirements for solvability. These requirements have been

initially formulated by Volterra (see e.g. Volterra [36] or Volterra and Peres [3?]), and for convenience are reproduced in an appendix. It is useful to note the following equality:

$$t_i(y) = \int_0^y 1/N\{\sum_{\substack{j=1\\ j\neq i}}^{N} \pi_j(p_j, P_j^{-1}(y)) - (N-1)\pi_i(p_i, P_i^{-1}(y))\} g(p|y)dp ;$$

From this equality, it becomes immediatly apparent that the regularity requirements (2) and (3), as formulated in the appendix, are satisfied for each of the function $t_i(.)$, $i=1,\ldots,N-1$. Furthermore, the assumptions (A3.2) and (A3.3) guarantee that $t_i(.)$ is N times differentiable in y for all i's. From (A3.2) we also conclude that (4) holds and from (A3.3) that (5) and (6) are satisfied.

Since all the requirements, formulated in the appendix, are satisfied, each of the equations (3.5i) will be solvable.(The equation might not be defined for all $y \in [0,Y_1]X \ldots X[0,Y_N]$. If this is the case, we can simply extend the functions. The method would the same as in chapter 1 theorem 1.3)

This concludes the proof.

Q.E.D.

It is worthwhile to examine this solution concept breaks down for a homogeneous oligopoly. For simplicity's sake, we only consider the case of a duopoly. We can argue either from a mathematical or from a heuristic perspective. Mathematically, for a homogeneous duopoly, the analog of equation (3.3_i) becomes:

$$\int_0^{\bar{P}(q_1+q_2)} T_i(p)f(p|q_1+q_2)dp \quad = \quad \int_0^{\bar{P}(q_1+q_2)} 1/2(\pi_j(p,q_j) - \pi_i(p,q_i))f(p|q_1+q_2)dp$$

Without extreme restriction on the inverse demand and the cost function, this equation is not solvable. To prove this, consider a change in the production vector which leaves the total output constant. The left-hand side of (3.51) cannot be affected by any such changes, since the output enters only as a sum into this integral. For the right-hand side this argument fails, because output enters non-linearly into the payoffs of the firms.

The essential difference between the homogeneous and the heterogeneous oligopoly is, that in the first case you can observe only one market price, whereas in the second case, you have an entire vector of prices upon which you can condition the transfers. For the homogeneous case, one can easily find a contract which satisfies the production incentive criteria. However, as we just proved, this contract would have to induce "dissipative " transfers (perhaps donations to hospitals or the purchase of an art collection!). Heuristically, we can easily explain this result. A production incentive contract needs to penalize a firm which overproduces. Unfortunately, in a homogeneous oligopoly, over production cannot be distinguished from a bad state of nature. Therefore, in a bad state of nature, you would have to punish everyone. This naturally leads to the sum of transfer being negative. Nevertheless, it would be

interesting to investigate this case as an alternative to the approach taken by Abreu, Pearce and Stachetti [2], Green and Porter [13] and Porter [33].

To conclude this section, we examine why it is heuristically reasonable that in a heterogeneous oligopoly, there exists an efficient contract. For simplicity, we only consider the case of a duopoly, though the heuristic could easily be extended to the case with N-firms. Suppose that for each firm, we examine the difference Δ between the expected price and the realized price. Also suppose that we agree that the firm with the larger difference must make a positive transfer to its competitor. Finally, let the transfer satisfy the requirement that, if neither firm cheats, the expected transfer of a firm is just zero. Consider now the problem of firm i which contemplates cheating on the agreement, but actually believes that j produces q_j^c. If i were to decide to produce beyond q_i^c, the probability of $(\Delta_i - \Delta_j) > 0$ would increase. This means that i would actually expect to lose some money to j. Theorem 1 simply states that, one can find a transfer function for which it is true that, (i) any change away from q_i^c would actually increase the expected transfer which firm i anticipates, by more than it expects to gain from cheating, and (ii) the negative of this function has a symmetric effect for firm j.

3.5 THE INFINITELY REPEATED GAME

The foregoing result is based on the assumption, that the firms

cannot deviate from an agreement previously reached. This requirement is very artificial in the framework of the last section. Indeed, once the outputs are produced and sold, and the output prices are observed, it is difficult to see what would prevent a firm, which is required by the mechanism, to make a positive transfer, to actually renege on its obligation. Clearly, in an environment in which cartels are illegal, we cannot assume that a central authority would enforce such a contract.

The purpose of this section is to show that, if we discard the critical assumption (A3.6) and repeat the remaining game infinitely, the firms will actually implement T in all periods - assuming that the discount rate is not too large. The argument will be based on the paper by Friedman [11]. At the expense of using more mathematics, the results can be generalized using optimal punishment, similar to Abreu [1].

Suppose the firms agree on a contract $\hat{T}$. Given any such agreement, a strategy σ_i^t for firm i, at time t will specify how much i will produce at the beginning of period t and, once the price vector P^t is observed, how much i will transfer. The strategy σ_i^t will depend on the history known to firm i at time t. We will denote this history H_i^t. In general, H_i^t will include; all of the past price vectors, all of the past transfer vectors and all of the past production of firm i. Given the contract $\hat{T}$, denote with $\sigma_I = (\hat{T};\ \sigma_i^1 \ldots)$ the strategy of firm i. Define the strategy σ_i^c as follows:

$$q^{c1} = \begin{cases} q_1^1 = q_1^c \\ T_1^1 = T_1^1(p) \end{cases}$$

and

$$\sigma_1^{ct}: = \begin{cases} q_1^t = \begin{cases} q_1^c & \text{if} \quad T^s = T^c(p^s) \quad \forall s,\ s < t \\ \cdot & \\ q_1^N & \text{otherwise} \end{cases} \\ \\ T_1^t = \begin{cases} T_1^c(P^t) & \text{if} \quad T^s = T^c(p^s) \quad \forall s,\ s < t \\ \\ 0 & \text{otherwise} \end{cases} \end{cases}$$

where $q^N = (q_1^N, \ldots, q_N^N)$ is the single period Nash and T^c is the contract defined by theorem 1. Using the same argument as in Friedman [11], we can show that, there exists discount rates for which the veotor of strategies $\sigma = (\sigma_1^c, \ldots, \sigma_N^c)$ is a Nash equilibrium. For $N \leq 3$, σ is exactly a non-cooperative equilibrium. For $N > 3$, there is a slight difficulty because the firms must decide how to allocate the transfers among themselves. We could make this decision part of the equilibrium strategy, but it is easy to see that it would lead to an infinite number of (equivalent) strategies. (The reader can easily convince himself that, if in some state there are more than two firms with a positive and more than two firms with a negative transfer, there is an infinite number of combinations of transfers between the firms, that leaves the total transfer of each firm constant). We

suppose that for $N \geq 4$, the firms come together once every period to harmonize their transfers. σ still remains essentially a non-cooperative equilibrium, since every firm could in any single period walk away from the cartel.

3.6 CONCLUSION

In this chapter we have shown, that under very weak restrictions and for some discount rates the combination of a heterogeneous oligopoly, an infinitely repeated game and the possibility of side payments guaranties that a cartel can sustain the monopolistic output as a non-cooperative equilibrium.

This result cannot be extended to a homogeneous oligopoly, though we believe that the method used in this chapter could also be applied for this case. It would provide a further explanation for dissipative expenditures like donations.

APPENDIX 1

Integral equation of the first kind are equations of the form:

$$\text{(A1)} \qquad \forall\, y \in [0,1], \qquad \lambda(y) = \int_0^1 \nu(x)k(x,y)\, dx$$

where $\nu(.)$ is the unknown. Equations of this type can be seen as a natural generalization of a system of n equations with n unknowns. When the kernel of the above equation satisfies the following requirement:

$$\forall\, y \in [0,1], \quad k(x,y) = 0 \quad \text{if} \quad y > x$$

the integral equation takes the particular form:

$$\text{(A2)} \qquad \forall\, y \in [0,1], \qquad \lambda(y) = \int_0^y \nu(x)k(x,y)\, dx$$

Equations of this type are known as Volterra equation of the first kind. The discrete analog of an equation of this type is an nxn system with a triangular matrix. It is well known that such a system is solvable if the elements along the diagonal of the matrix are non-zero. We would expect that this result generalizes to the continuous case. Volterra was the first to show that it is indeed the case (Volterra [36,38]) if some regularity requirements are satisfied. (A2) is solvable if the following conditions are satisfied:

(i) $\lambda(0) = 0$,

(ii) $\forall\, x,y \in [0,1]$, $\lambda(x)$ and $k(y,x)$ are differentiable in x,

(iii) $\forall\, x \in [0,1]$, $k(x,x) \neq 0$,

(iv) $\forall\, x,y \in [0,1]$, $k_x(y,x)/k(x,x)$ is bounded.

(i) is just a consistency requirement, since for $x = 0$ the integral is always zero.

APPENDIX 2

Suppose that $x = \theta + \tau$, where θ and τ are two independent non-degenerated random variables. Define $z(x) = -\exp(-x)$. Then

$$z(\theta) = -\exp(-\theta)E(\exp(-\tau))$$

This is an increasing, concave function and assumption 2 is satisfied. In this appendix, we also want to prove that if $f(x|\theta)$ is a <u>regular</u> density of the exponential family assumption 2 is satisfied[7]. Conditional densities of the exponential family are of the form:

$$f(x|\theta) = a(\theta)b(x)\exp(-\alpha(\theta)\beta(\theta)) \qquad \text{for } x \in [x_0,x_1],\quad \theta \in [\theta_0,\theta_1]$$

A density function of the exponential family is said to be <u>regular</u> if the following requirements are satisfied:

i) $b(x)>0$ for $x\in[x_0,x_1]$, $a(\theta)>0$ for $\theta\in[\theta_0,\theta_1]$

ii) $\beta(.)$ is differentiable and increasing over its range of definition

iii) $\alpha(.)$ is positive and increasing over its range of definition

iv) $(0,\infty) \subset (\beta(x_0),\beta(x_1))$

As mentioned earlier these conditions are satisfied by the normal, log-normal, Beta and Gamma distributions if θ is a shift parameter that affects the distribution according to the assumption made above. We want to show that assumption 2 is satisfied for densities of the above type. For any function $z(x)$, we have:

$$\dot{z}(\theta) = \int_{x_0}^{x_1} z(x)a(\theta)b(x)\exp(-\alpha(\theta)\beta(x))dx$$

$$= \int_{\beta(x_0)}^{\beta(x_1)} \{z(\beta^{-1}(y))a(\theta)b(\beta^{-1}(y))\exp(-\alpha(\theta)y)/\beta'(\beta^{-1}(y))\}dy$$

Let us choose the function z(.) such that:

$$z(\beta^{-1}(y))b(\beta^{-1}(y))/\beta'(\beta^{-1}(y)) = \begin{cases} -y^n & \text{if } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

If we now substitute this into the integral we obtain:

$$\dot{z}(\theta) = -a(\theta)\int_0^{\infty} \exp(-\alpha(\theta)y)y^n dy$$

The integral is a Laplace transform of the function y^n. Using any standard table we obtain:

$$\dot{z}(\theta) = -a(\theta)n!/(\alpha(\theta))^{n+1}$$

Given the restriction on $\alpha(.)$ it is immediate that for n large enough $\dot{z}(.)$ is increasing and concave.

APPENDIX 3

Volterra equations of the first kind with multiple integrals are a straightforward generalization of the types of equations examined in the first appendix. A Volterra equation of the first kind with multiple integrals is defined as follows:

$$V\ (y_1, y_2, \ldots, y_n) \in \mathop{X}_{i=1}^{n} [0, Y_i]$$

$$\text{(A3)} \quad \lambda(y_1, \ldots, y_n) = \int_0^{Y_1} \cdots \int_0^{Y_n} \nu(x_1, \ldots, x_n) k(x_1, \ldots, x_n, y_1, \ldots, y_n) dy_1 \ldots dy_n$$

where $\nu(.)$ is the unknown function.

Using the result from Volterra [37] (alternatively, see Volterra and Peres [38]) (A3) will be solvable if the following conditions are satisfied:

(i) $\lambda(.)$ and $k(.)$ are n times differentiable in the variables y.

(ii) $\lambda(y_1, \ldots, y_n) = 0$ if $y_i = 0$, $i=1, \ldots n$;

(iii) $\forall\ k = 1, \ldots, n-1$

$$\frac{\partial^k \lambda}{\partial y_{i_1} \ldots \partial y_{i_k}} (y_1, \ldots, y_n) = 0 \quad \text{if } y_j = 0,\ j=1, \ldots n,\ j \neq i_1, \ldots, j \neq i_k$$

(iv) $k(y_1, \ldots y_n, y_1, \ldots, y_n) \neq 0$ for all possible $(y_1, \ldots, y_n)$.

(v) $k(x_1, \ldots, x_n, y_1, \ldots, y_n)/k(y_1, \ldots, y_n, y_1, \ldots, y_n)$ is bounded.

As in appendix 1, the conditions (ii) and (iii) are just

consistency requirements. Indeed, under the conditions of (ii) and (iii) the integrals on the right hand side will all be zero.

REFERENCES

[1] Abreu, D. (1986): "External Equilibria of Oligopolistic Supergames", Journal of Economic Theory.

[2] Abreu, D., Pearce, D. and Stacchetti, E. (1986): "Optimal Cartel Equilibria with Imperfect Monitoring", Journal of Economic Theory.

[3] Arrow, K.J. (1985): "The Economics of Agency" in J.W. Pratt and R.J. Zeckhauser, eds., Principals and Agents (Cambridge, Mass.: Harvard Business School Press).

[4] Baron, D. and Myerson, R.B. (1982): "Regulating a Monopolist with Unknown Costs", Econometrica.

[5] Christensen, J. (1982): "Communication in Agencies", Bell Journal of Economics.

[6] Cournot, A. (1897): "Researches into the Mathematical Principles of the Theory of Wealth", New-york, MacMillan.

[7] Crémer, J. and McLean R. (1985a): "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," CARESS Working paper No. 85-15, University of Pennsylvania.

[8] Crémer, J. and McLean R. (1985b): "Optimal selling strategies under Uncertainty for a Discriminating Monopolist when demands are Inter-dependent", Econometrica.

[9] Dupuit, J. (1844): "De la Mesure de l'Utilité des Travaux Publiques", Annales des Ponts et Chaussées.

[10] Fredholm, I. (1903): "Sur une Classe d'équation fonctionelles", Acta Mat.

[11] Friedman, J. (1971): "A Non-cooperative Equilibrium for Supergames", Review of Economic Studies.

[12] Gjesdal, F. (1976): "Accounting in Agencies", Mimeo, Graduate School of Business, Stanford University.

[13] Green, E.J. and Porter R.H. (1984): "Non-cooperative Collusion under Imperfect Price Information", Econometrica.

[14] Hart, R. and Moore, J. (1988): "Incomplete Contracts and Renegotiation", Econometrica, to appear.

[15] Holmström, B. (1979): "Moral Hazard and Observability", Bell Journal of Economics.

[16] Holmström, B. and Milgrom, P. (1985): "Aggregation and Linearity in the Provision of Intertemporal Incentives", Econometrica, to appear.

[17] Hotelling, H. (1938): "The General Welfare in to Problems of Taxation and of Railway and Utility Rates", Econometrica.

[18] Laffont, J.J. and Tirole, J. (1986): "Using cost-observation to Regulate Firms," Journal of Political Economy.

[19] Laffont, J.J. and Tirole, J. (1985): "Auctioning Incentives Contracts", mimeo, MIT.

[20] MacDonald, G.M. (1984): "New Directions in the Econmic Theory of Agency", Canadian Journal of Economics.

[21] Maskin and Riley (1984): "Monopoly with Incomplete Information", Rand Journal of Economics.

[22] Maskin, E. and Riley, J. (1980): "Auctions Design with Correlated Reservation Values," Mimeo.

[23] McAfee, P. and McMillan, J. (1987): "Auctions and Bidding", Journal of Economic Literature.

[24] McAfee, P. and McMillan, J. (1987): "Competition for Agency Contracts", Rand Journal of Economics.

[25] McAfee, P. and McMillan J. (1987): "Bidding for Contracts: A Principal-Agent Analysis," Rand Journal of Economics.

[26] McAfee, P., McMillan, J. and Reny, P. (1988): "Extracting the Surplus in Common Value Auctions", Econometrica, Forthcoming.

[27] McAfee, P. and Reny, P. (1988): "Correlated Information and Mechanism Design", Mimeo, The University of Western Ontario.

[28] Melumad, N. and Reichelstein, S. (1987): "Value of Communication in Agencies",mimeo, G.S.B., Stanford University

[29] Mirrlees, J. (1976): "The Optimal Structure of Incentives and Authority within an Organization", Bell Journal of Economics.

[30] Myerson, R.B. (198[illegible]): "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems", Journal of Mathematical Economics.

[31] Myerson, R.B. (1979): "Incentive Compatibility and the Bargaining Problem", Econometrica.

[32] Picard, E. (1909): "Sur les Equations Integrales de Premières espèces", Cahier de Recherche de l'Academie des Sciences, Paris.

[33] Porter, R.H. (1983): "Optimal Cartel Trigger Price Strategies", Journal of Economic Theory.

[34] Riordan, M.H. and Sappington, D.(1987): "Awarding Monopoly Franchise", American Economic Review.

[35] Sappington, D. (1984): "Incentive Contracting with Asymmetric and Imperfect Precontractual Knowledge", Journal of Economic Theory.

[36] Volterra, V. (1896): "Sulla Inversione Degli Integrali Multipli", Rend. Ac. Lincei, serie 5, vol. 5.

[37] Volterra, V. (1896): "Sulla Inversione Degli Integrali Definiti", Atti Ac. Torino. vol. 31.

[38] Volterra, V. and Peres, J. (1936): "Theorie Generales des Fonctionelles", Gauthier-Villars, Paris.